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Teaching To The Test, . . . Sort Of

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1. Introduction

To enroll in college level mathematics courses freshmen entering the California State University System must demonstrate "mathematical readiness," through one of the following criteria:

(1) SAT Math Reasoning Test: > 550,
(2) ACT Math: > 23,
(3) AP Math: > 3,
(4) Early Assessment Program (EAP): Exempt status, [1]
(5) College Course: C or better,[2]
(6) Entry Level Mathematics (ELM) Examination: > 50.

Simply put: "Readiness" is essential to success in collegiate mathematics and students who meet none of these qualifications are required to take essentially high school mathematics remedial courses at the university! Far from academically ideal and very expensive for taxpayers of California.

This article addresses a possible solution to improve the chances of those students who have none of the first five bypasses to qualify through (6), the ELM.

In-house research done at California State University, Long Beach (CSULB) shows that of the incoming freshmen taking the ELM, a population of students with an average B+ high school GPA, nearly half score below 50. [3] This is likely

[1]The California State University (CSU) requires high school students to take the English Placement Test (EPT) and the Entry Level Mathematics (ELM) exam prior to enrollment in the CSU unless they are exempt by means of scores earned on other appropriate tests such as the CSUs Early Assessment Program (EAP) tests in English and Mathematics, the SAT, ACT, or Advanced Placement (AP). See: CSU url on Admissions and Records.

[2]ibid

[3]Brown, C. "Analysis of ELM scores and GPA, California State University, Long Beach." 2005. McNair Scholarship Student Research Project. (unpublished.)

the case at many CSUs. As a result, at CSULB thousands of incoming students are channeled through courses equivalent to high school algebra: MAPB 1, MAPB 7, and MAPB 11 (MAth Pre Baccalaureate). This must be burdensome on University fiscal and personnel resources; as important, this is a significant time delay for students who pursue a baccalaureate, extending the time a four to five year academic program significantly.

Central to this article is the "Practice for ELM," sponsored by the Chancellor's Office of the California State University system, a program to be established for high school students to deal with this problem.

2. Background and Program Description

To assist high school students who will take the ELM qualify for college level courses, the California State University Chancellors Office contracted this author to develop an online practice test at for the CSU "Math Success" website where high school students could use to gain more mathematics sophistication, and yes, practice for passing ELM. See Figure 1.

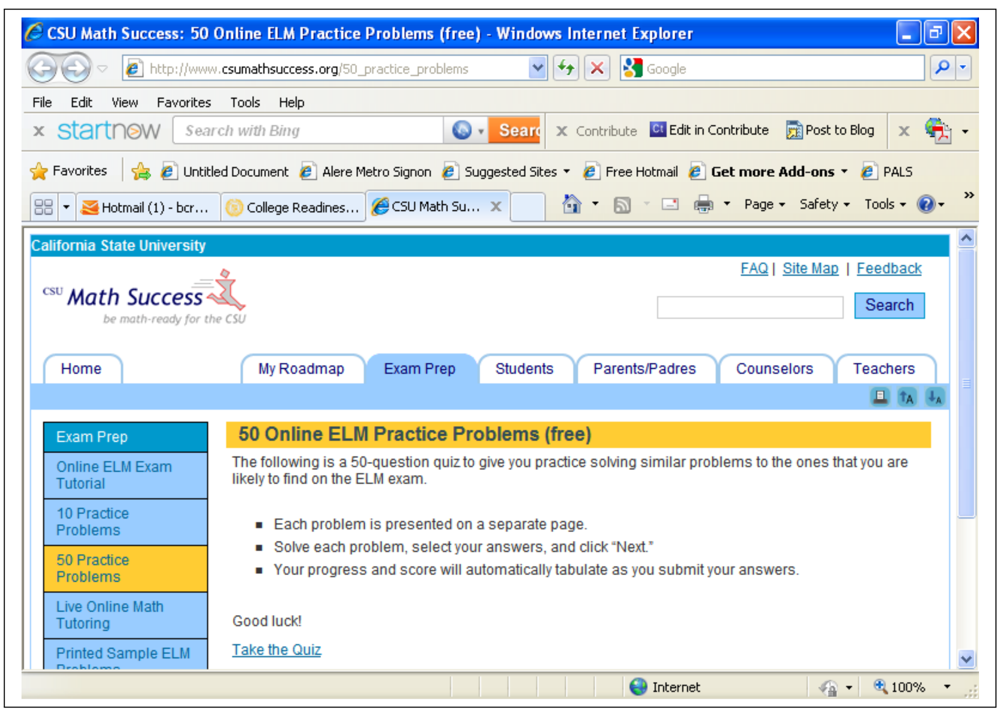
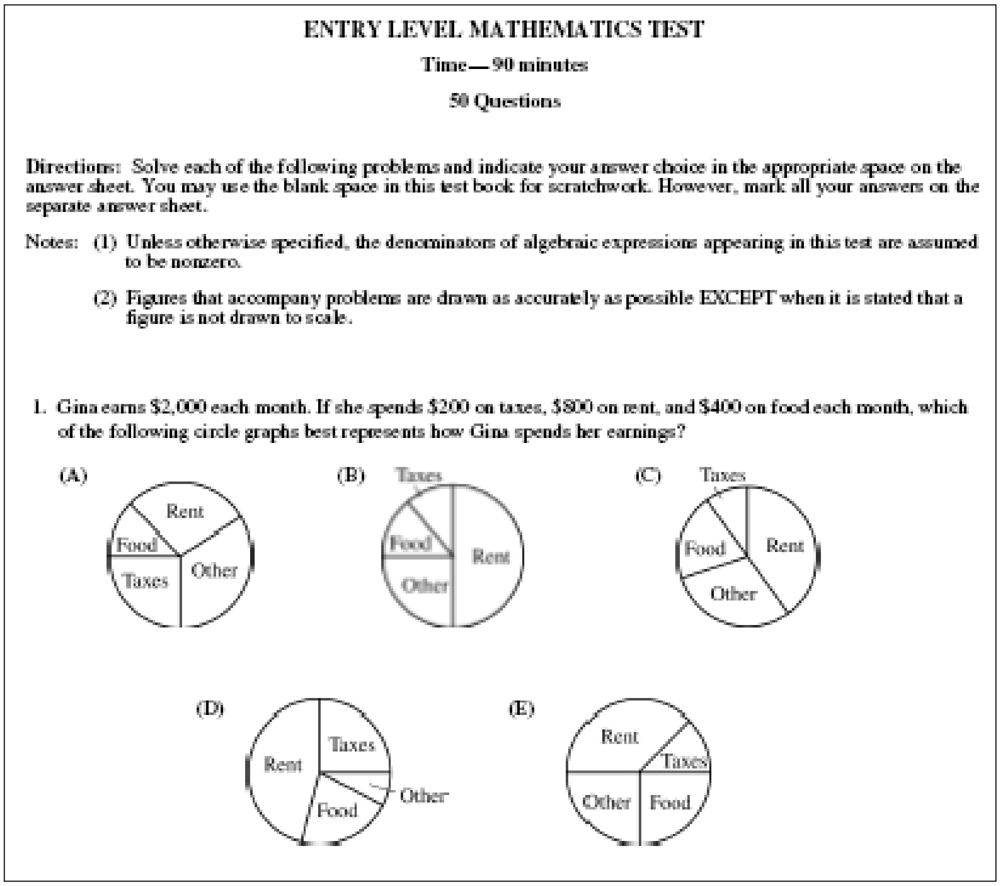


Figure 1. ELM Practice Exam Website

The "quizzes" on this site consist of released ELM test questions used in previous ELM tests. Figure 2 displays a typical question.



ENTRY LEVEL MATHEMATICS TEST

Time — 90 minutes

50 Questions

Directions: Solve each of the following problems and indicate your answer choice in the appropriate space on the answer sheet. You may use the blank space in this test book for scratchwork. However, mark all your answers on the separate answer sheet.

Notes: (1) Unless otherwise specified, the denominators of algebraic expressions appearing in this test are assumed to be nonzero.

(2) Figures that accompany problems are drawn as accurately as possible EXCEPT when it is stated that a figure is not drawn to scale.

1. Gina earns $2,000 each month. If she spends $200 on taxes, $800 on rent, and $400 on food each month, which of the following circle graphs best represents how Gina spends her earnings?

Figure 2. Typical ELM Practice Quiz Question

I found that this platform is amenable for “teaching to the test,” but in a positive fashion. I use it to teach the mathematics concepts high school students need to qualify for college level mathematics courses. The online practice familiarizes students with the various needed skills and concepts and, yes, also focus on the types of problems students will encounter on the ELM. Thus in a way we are “teaching to the test.” But the caveat is important: each wrong choice of answer on a given test item is accompanied by what we could label an “error analysis” that explains to the student *why* that particular choice is not correct. This is far from a face-to-face explanation, but worth the effort, I think.

Research on multiple choice tests indicates that wrong answers, “distracters,” are designed carefully by test writers to expose the subject’s conceptual and methodological errors, reflecting as much as possible the student’s competence in the material itself, and emphatically not, the student’s test taking skills. This is an unfortunate interdependence. Ideally, a test should expose the student’s weakness in the subject matter, not test taking skills.

I use the metaphor of “tip of an iceberg” to explore students’ hidden misconceptions when looking for an explanation to wrong answers; the hidden mass of the iceberg. Suggestions in the answer part of the program described in the figures above are based the author’s educated guess of what has gone wrong.

In descending order of frequency, I found the following to be the possible “below the surface” reasons for wrong answers:

(1) The student misunderstood or misinterpreted the problem (and the answer would have been correct for that interpretation).
(2) The student made an incorrect calculation or algebraic faux-pas.
(3) The student made a conceptual error.
(4) The student made a careless error.
(5) The student guessed at the answer.

There is a limited number of suggestions to the students for wrong choices “4. The student made a careless error” and to a lesser extent “ 5. The student guessed at the answer”. These have to do with study habits, and are a challenge to “repair” in any teaching-learning situation. Thus the first three reasons are the ones the program concentrated on.

“1.The student misunderstood or misinterpreted the problem (and the answer would have been correct for that interpretation)” is a balance between communication skills and mathematics, perhaps more the former than latter: EL comes to mind here.

Thus the “Teaching to the test” challenge here was advising students who had made a conceptual or skills error. The responses to wrong answers were guided mostly by these two reasons for an incorrect answer.

3. Teaching *Using* the Test

Our intent was to give meaningful feedback to students taking the practice test. For each wrong answer I concentrated on interpreting the student’s thinking, retracing the flaw I thought I detected, and providing an appropriate hint or suggestion that might be helpful.

This is the meaning of the title “Teaching to the test . . . sort of.” It is more a tutorial program than a teaching program. As such, the authors tried to “get inside the student’s head” to discover where the mental process might need a road sign. No answers were given outright, just hints and suggestions to steer the student in the right direction; better *a* right direction.

There were fifty problems in the practice ELM, each with four distracters, so there were scores of possible incorrect solutions. The challenge to the authors was to identify “the left turn” or to use our metaphor, the bulk of the iceberg below the surface that represented the incorrect thinking, and use the hint do what was needed to make the correct choice on the next attempt. If the student chose another distracter, a different hint was given. Even if our hint did not exactly reflect the student's mental process, the student could compare and contrast his or her mental process with the process in the feedback.

4. Examples

Different questions and their distracters on the practice test revealed different shortcomings in a student's conceptual understanding and procedural skill.

For the sample question in Figure 2 we display below two types of responses. Figures 3 and 4 were the hints the authors planned. If the student picked distracter A, only the hint shown in the top half of Figure 3 would appear. Similar feedback is given for distracters B, D, and E in Figure 4. An approval statement was presented to support the correct answer C.

The reader will appreciate that this innocent looking problem is not so innocent; the student needs to know his or her geometry and use proportional reasoning.

Figures 5, 6, 7, and 8 offer a short explanation in the legend part of the figure.

Note: Each of the next six pages contains a single screen shot that takes up the entire page. This was done to make the screen shots more legible.

(A)

Note that the sum of the expenses for taxes and rent is $1000, which is *half* her monthly income. It is hard to say that (A) shows the sum of taxes and rent is half the circle graph. See table below

Gina's Monthly Budget			
Type of Expense	Amount	Approximate Percent	Degrees
Taxes	$200	10%	36
Rent	$800	40%	144
Food	$400	20%	72
Other	$600	30%	108
Total	$2,000	100%	360

(B)

Gina's monthly rent expenses, $800 plus taxes is half her salary, but (B) shows it to be more than half the circle graph. See table below.

Gina's Monthly Budget			
Type of Expense	Amount	Approximate Percent	Degrees
Taxes	$200	10%	36
Rent	$800	40%	144
Food	$400	20%	72
Other	$600	30%	108
Total	$2,000	100%	360

FIGURE 3. One possible explanation for choosing the wrong answer A or B to the problem in Figure 2.

(C)

Yes, Gina's monthly rent expenses, $800, plus her $200 for taxes adds $1,000, half her salary, half the circle graph.

(D)

Gina's monthly rent expenses, $800 plus taxes is half her salary, but (D) shows it to be more than half the circle graph. See table below.

Gina's Monthly Budget			
Type of Expense	Amount	Approximate Percent	Degrees
Taxes	$200	10%	36
Rent	$800	40%	144
Food	$400	20%	72
Other	$600	30%	108
Total	$2,000	100%	360

(E)

Gina's monthly rent expenses, $800 plus taxes is half her salary, and that is true in (E) but her "other" expense is $600 and $\frac{600}{2000} = 30\%$ of the circle graph, not 25%. See table below.

Gina's Monthly Budget			
Type of Expense	Amount	Approximate Percent	Degrees
Taxes	$200	10%	36
Rent	$800	40%	144
Food	$400	20%	72
Other	$600	30%	108
Total	$2,000	100%	360

FIGURE 4. One possible explanation for choosing the wrong answers D or E to the problem in Figure 2. Answer C is correct.

3. $0.215 - 16.215$

(A) 16.43

You may have confused the rule for subtracting signed numbers by changing the sign of -16.215 and adding to get $0.215 - (-16.215)$? This is not the case here, it is simply a subtraction problem like $3 - 10 = -7$. Or, you might have thought that we cannot subtract a larger number from a smaller one. Indeed we can subtract any two real numbers.

(B) 16

You subtracted OK, but "in the wrong direction." Note that the second number is negative and it is larger in absolute value than the first number, so the difference should be negative.

(C) -14.065

$-14.065 = 2.15 - 16.125$. So your idea was correct, but you miscopied the 0.215 as 2.15. Try it again with 0.215.

(D) -16

Correct.

(E) -1643

Looks like you added: $(-0.215)+(-16.215) = -16.43$. This is a subtraction problem: $+0.215-(+16.215)$. Try it again.

FIGURE 5. ELM Practice Exam Problem 3 on number sense.

10. Leni spends $\frac{1}{3}$ of her income on rent and $\frac{1}{4}$ of her income on car expenses. What fraction of her income is left for other expenses?

(A) $\frac{1}{2}$

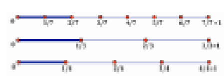
Keep in mind that 1 represents Leni's total salary, $\frac{1}{2}$ half her salary, and $\frac{1}{4}$ one-fourth her salary. Now $\frac{1}{3}+\frac{1}{4}$ is *about* $\frac{1}{2}$ but it is really $\frac{7}{12}$. You can think in terms of percent, if that is easier for you, then 1 is equivalent to 100%

(B) $\frac{1}{5}$

You started off OK by trying to add the fractions $\frac{1}{3}+\frac{1}{4}$, but you got $\frac{1}{5}$. Should be $\frac{7}{12}$, which then is subtracted from 1 to get the right answer. Please note: Perhaps you did not add the fractions correctly; we cannot simply add numerators and denominators. We need to first find a common (unit) denominator. Do not add fractions like this: $\frac{1}{3}+\frac{1}{4}=\frac{1+1}{4+3}$. This is bogus which makes $=\frac{2}{4+3}=\frac{1}{2+3}$ bogus. (We cannot cancel the 2's . . .why?) $=\frac{1}{5}$, which is the wrong answer. Should be $\frac{7}{12}$.

(C) $\frac{5}{7}$

You started off on the right foot by trying to add $\frac{1}{3}+\frac{1}{4}$. But you added the numerators and denominators, and then somehow got $\frac{1}{3}+\frac{1}{4}=\frac{1+1}{4+3}=\frac{2}{7}$. But this is not correct, nor is it the way to add fractions. Also, $\frac{1}{3}+\frac{1}{4}\neq\frac{7}{12}$. When adding fractions we cannot simply add numerators and denominators. The picture below shows you that $\frac{1}{3}+\frac{1}{4}$ cannot be $\frac{7}{12}(\approx\frac{1}{2})$. . .



(D) $\frac{5}{12}$

Correct.

(E) $\frac{11}{12}$

Looks like you did this: $\frac{1}{3}+\frac{1}{4}=\frac{1\times1}{3\times4}=\frac{1}{12}$. This is not the right way to add fractions. We cannot simply add numerators and denominators. We need to first find a common (unit) denominator. Do not add fractions like this: $\frac{1}{3}+\frac{1}{4}=\frac{1+1}{4+3}$. This is bogus which makes $=\frac{2}{4+3}=\frac{1}{2+3}$ bogus. (We cannot cancel the 2's . . .why?) $=\frac{1}{5}$, which is the wrong answer. Should be $\frac{7}{12}$.

FIGURE 6. ELM Practice Exam Problem 10; the student could make incorrect calculations and make conceptual errors in how fractions are added. Appropriate responses are given.

18. $\frac{(x-1)(3x+6)}{(3x-3)}$

(A) $x+2$

The idea is to simplify the numerator and denominator by factoring if we can to see if the numerator and denominator share common factors, which can then be divided out (or as is said more commonly, "cancelled.") Example: $\frac{14}{21} = \frac{2\times 7}{3\times 7} = \frac{2}{3} \times \frac{7}{7} = \frac{2}{3} \times 1 = \frac{2}{3}$. Here $\frac{(x-1)(3x+6)}{(3x-3)} = \frac{(x-1)(3)(x+2)}{3(x-1)} = \frac{3(x-1)(x+2)}{3(x-1)} = \frac{(x+2)}{1} \times \frac{3(x-1)}{3(x-1)} = \frac{x+2}{1} = x+2$ if $x \neq 1$.

(B) $x+6$

Looks like you did everything right except factoring $(3x+6)$; you got $3(x+6)$ instead of $3(x+2)$. Try it again

(C) $2(x-1)$

Looks like you wanted to "cancel" the $(3x+6)$ by $(3x-3)$ to get 2. (And ignore the minus.) If so, then your answer would be $2(x-1)$. BUT this is not right. Try factoring 3 from both $(3x+6)$ and $(3x-3)$.

(D) $-(2x-1)$

Looks like you wanted to "cancel" the $(3x+6)$ by $(3x-3)$ to get -2, but this is not right. Try factoring just 3 from both $(3x+6)$ and $(3x-3)$ to get $\frac{3(x+2)}{3(x-1)}$.

(E) $\frac{(x-1)(x+6)}{(x-3)}$

Looks like you wanted to "cancel" just the 3 from the 3x in $(3x+6)$ and the 3x in $(3x-3)$ to get $(x+6)$ and $(x-3)$, but this is not right. When you factor $\frac{(3x+6)}{(3x-3)}$ factor the 3 from both $(3x+6)$ AND $(3x-3)$ to get $\frac{3(x+2)}{3(x-1)}$.

FIGURE 7. ELM Practice Exam Problem 18 is an algebra problem.

The distracters tried to expose algebra errors. And again, for each answer, the authors provide a hint that may clarify students' thinking.

Age(years)	Percent of Students
22 or younger	44.7
23 - 25	25.2
26 - 35	20.1
36 or older	10.0

13. The table above shows the percent of students in different age groups at a university campus. The total number of students at the campus is 8,987. Approximately how many of the students at the campus are of age 25 years or younger?

(A) 2300

Looks like you used 25.2%. This represents *only* 23 to 25 year olds. We want *all* 25 year olds or younger; this includes the "22 or younger group," so we want 44.7% + 25.2% = 69.9% or about 70% of the 8987 students. Imagine all 8,987 students in the football stadium. The 22 years or younger sit in the stands on one side of the field, the 23, 24, and 25 years old sit in the stands opposite the first group, the 26 to 35 year olds in the stands behind one end zone, and the 35 years old and older on the opposite end zone. Which group is the largest? If 100% represent the 8987 students, then 44.7% + 25.2% = 69.9% or about 70%.

(B) 2700

Looks like you found the percent of students OLDER than 25: 20.1% + 10% = 30.1% and 2700 us about 30% of 8987. Now try to find the number of students YOUNGER than 25. You can rescue the situation by subtracting . . . or multiplying.

(C) 4,000

Looks like you computed the percent of students 22 years old or younger. 44.7% x 8987 = 4,017. But what about the 23 to 25 year olds? You need to add the two percentages and then multiply.

(D) 5,000

Looks like you found the percent of students OLDER than 22. You added 25.2% + 20.1% + 10% = 55.3% and 55.3% of 8987 is about 5,000. We want just the opposite: all students YOUNGER than 22 years old.

(E) 6,300

Good. 44.7% + 25.2% = 69.9% or about 70%, so 0.70 x 8987 is about 6300.

FIGURE 8. ELM Problem 13 involves interpretation.

The student could make interpretation errors. The problem is complex enough that the student could get confused in which process should be pursued. As with the other problems, each answer, wrong or right has a response that is hopefully helpful.

5. Conclusion

The research the authors did on the possible reasons for the wrong answers on the ELM practice test was interesting and hopefully helpful to students and insightful for teachers and student. Professional development for mathematics teachers could include analysis of distracters on multiple choice questions followed by plausible though processes leading to them. Another possible application of this approach is to help students develop their self-awareness. The feedback helps them think about what they did. The development of self-awareness is important in educational maturity, and the essence of mastering mathematics is correcting one's mistakes.