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Inductive Math Instruction

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It is no secret that America no longer leads the world in math achievement. Efforts over the last 15 years, focusing on the integrity of instruction, have assured that every student is taught the correct material, by a qualified teacher, with a standardized textbook bearing the state's seal. Presumably, as the correct materials assembled in the correct fashion will build a sturdy house, solid instruction and instructional materials should produce a solid education. Now, with over a decade of standardized curriculum and assessment behind us, these measures have not borne the results expected. The meager gains in American test scores have been doubled and tripled by Asia and Europe. New classroom equipment aside, we must look beyond the cover, to our approach to education and the underlying philosophy of learning.

Educational policy is the epistemological offspring of Jean Piaget's developmental stage theory, a model charting children's cognitive development through distinct stages of concrete and abstract thought. This gospel of age-appropriate curriculum dictates what should be learned when, that young children should study arithmetic with concrete objects while adolescents enter the abstract realm of algebra. But how does learning happen? What is learning? What physiologically underlies it? For insight into this most basic, yet ineffable topic we turn to the theory of constructivism, modern neurology, and the wisdom of children.

Constructivism, a revolutionary paradigm of learning, asserts that it is the learner who must construct meaning, who is the architect of the knowledge. This theory (and philosophy) was founded by Piaget and reinvented by many, each highlighting different aspects. Focusing on the impetus of learning, Lev Vygotsky emphasized the roles of language and social interaction: meaning is constructed by expressing one's ideas in language and collaborating with other people. In his most divergent and dramatic contribution, Vygotsky also believed that experience informs development. From Vygotsky's perspective, the standard classroom model – the teacher actively disseminating material to passive students as if loading information into a computer – is upside down. Since students must ultimately teach themselves, a teacher can only light the way along their journey. Changing the emphasis from teaching to learning, Vygotsky's work promotes students to center stage, as the star players in their learning.

My first year of high school was met with an unexpected math book: College Preparatory Mathematics (CPM). Twenty years later, I remember learning the Pythagorean Theorem like it was yesterday. This chapter began not with the statement but with a project – an application that in most textbooks would be banished to the back of the chapter, the repository untouched by teachers. The lesson began on the next page with an investigation of right triangles, measuring their sides and recording the data in a table. I noticed that the squares of the shorter sides combine to the square of the longest side. Concluding the investigation, we stated a hypothesis, putting into words the truth unveiled: the Pythagorean Theorem. Embodying Vygotsky's paradigm, the experience preceded the learning. To culminate the chapter, we executed the anticipated project, calculating the length of a guy wire needed run from the corner of a roof to the top of an antenna, putting to use the new tools developed over the chapter. I built a three-dimensional model of the structure described.

Sadly, the novel pedagogical approach was not appreciated by everyone. A favorite subject of parent complaints, CPM was condemned by the board and eventually thrown to the wayside in favor of a more traditional program, the kind the parents wanted, the kind that had not worked for them... Though CPM's life was short at Sonora High School, its spirit lives on in me as a teacher.

CPM's design introduces a topic with specific examples and leads the students to notice patterns and infer general truths. This is an approach I now know as inductive. A typical textbook, in a direct instruction format would announce the Pythagorean Theorem by stating $a^2 + b^2 = c^2$ and follow with specific examples worked out for the student. The jargon of educational theory names many variations of inductive teaching (and inductive learning on the part of the student). Inquiry highlights the questioning aspect used by a teacher. Questioning is a motivating tool and is required of the students as critical thinking in tasks (National Institute – Landmark College, 2005). A lab using the scientific method, requiring students to observe data and form hypotheses, is a quintessential inquiry lesson. Discovery learning provides the student with not the answer but with the means to find it. The learning cycle may be to inductive teaching as the five step lesson plan is to deductive teaching. Inductive teaching, requiring the learner to construct meaning, is inherently constructivist. To classify a lesson as a particular nuance of inductive teaching is not a goal of this article, but most lessons discussed involve inquiry and discovery components. In Vygotsky's vein, cooperative learning often accompanies inductive teaching, as with CPM.

As a graduate teaching assistant, constructivism was not how my teaching began. Like most college instructors, I lectured the entire class period, save a few minutes to answer questions. The students appreciated explicit definitions, numbered theorems, and sequences of steps to rigidly follow presented like street directions. One student said, "Your notes are poetry." I was praised on my evaluation for "thoroughly covering the material." Fortunately, the next educational setting, with a very different educational philosophy, would reformat my teaching.

In my first year of teaching eighth grade algebra at a charter school, I was asked to lecture less. The charter school coordinator enlightened me that eighth graders, though quiet they may be, cannot absorb a 40 minute lecture. Little did I know, starting my second year with more direct instruction than ever that – like a caterpillar entering a cocoon – I would emerge in May a different teacher.

That year blessed me with an unusually heterogeneous group of students. Also, I became very interested in plants. Two seventh graders, having advanced to algebra 1 a quarter into the school year, were always finished early. Not wanting them to feel bored (and cause problems), I let them work ahead in the curriculum and was amazed by what they could do without a lesson. They spent most of the time working together, figuring out the math, helping each other put together the pieces. Coming over to them occasionally, I would answer a question or give them a nudge in the right direction. My guidance was still crucial, but they were in the driver's seat. As these two students became the workers laying down the bricks, I became the inspector overseeing a sound construction.

Soon this seed, a new instructional model, grew into my other classes. I began to group students working on the same topic to rotate around the room, providing short bits of instruction to keep each group moving as they put together the pieces. In one week my classroom had morphed into the learning center model, befitting of a primary classroom, and I had become a CPM teacher. At the beginning of the year, the students answered my questions. At the end of the year, I answered their questions. They were in the driver's seat. The state test scores jumped this year by leaps and bounds. My construct of learning was changing – from that of the teacher building to a new theme of the students growing. I now "tended to" the students, like the new plants in my office, nurturing their growth. Neuroscientists, now peering inside the brain as learning happens, support a plant growth model, in a very literal sense, as the basis of learning.

While direct instruction is still essential, I have grown to favor inductive teaching. I begin my unit on slope and linear equations with boards and tape measures. I place the boards along the wall, a short and a long board at a shallow angle and another pair at a steep angle. I challenge the students with an inquiry: devise a metric, a way to measure the steepness of the boards. The shallow long board and steep short board help them realize that neither the height along the wall nor the distance to the wall along the floor is sufficient; they must find the ratio. The goal to obtain a large number for the steep boards and a small number for the shallow boards, the students are forced to use the rise as the numerator and the run as the denominator. Patterned after CPM's investigation of right triangles, my unit on linear equations begins with graphing different equations: $y = x$, $y = 2x$, $y = 3x$, etc. Then students discover the coefficient as the slope. I then have them predict the effect of the following change: $y = x + 3$, $y = 2x + 3$, $y = 3x + 3$. The students discover the math and tell me the theorem. From specific examples, students make the generalization that the graph of $y = mx + b$ is a line with slope m and $y-$intercept b. This theorem is the students' result. In contrast, a direct instruction approach to this topic would begin with the theorem as the teacher's opening statement and move on to specific examples.

Even after K-12 transformation, I did not think that the condensed pacing at the college would afford any modality other than lecture sessions followed by a few practice problems, or that college students would want a student-centered model. For the last two years at the college, I have gravitated toward an inductive instructional model, reducing my lecture time by more than half. No longer preoccupied with covering every minute detail and case the students will encounter, I leave more for them to construct. Most of my recent classes have achieved much higher test scores, scoring particularly better on problems that require levels of learning higher on Bloom's Taxonomy. Last summer roughly half of my algebra 1 students mastered distance-rate-time and mixture problems, a huge improvement from the past success rate of roughly ten percent. Student evaluations support the change, as summarized in the table below.

Preference	Number of students
Support having less lecture	13
Would have preferred more lecture	3
*Support inquiry-based instruction	4

*The data were limited for these criteria.

Students wrote the following comments:

> *Student 1:* Most people learn by doing, and I think it's crucial to do problems in class. That way you [the teacher] can teach them how to do the problem correctly, versus lecturing, which becomes <u>very</u> <u>boring</u>.

> *Student 2:* I believe it was very beneficial to spend more time in class on problems, as it gives us a chance to give feedback and collaborate with classmates.

My biggest surprise is that inductive teaching has been not only more effective, but also more efficient. For the last two years I have been ahead of my pacing plans and able to cover nearly twice the material. How could this be? First, less time spent on lecture gives students more time to learn. Second, inquiry-based teaching, by using central underlying concepts to drive knowledge and procedures, unifies the curriculum.

By applying the meaning of exponents, my algebra students infer the product rule: five factors of x multiplied by three factors of x reassociates to eight as the power on x. Covering the quotient rule – the next section – in the same lesson, I ask the students to consider $\frac{x^5}{x^3}$, which by the same reasoning leaves two factors of x or x^2. Breaking the tradition of teaching slope-intercept form and point-slope form as two completely different things, I presented them this year as one and the same. Slope intercept form, $y = mx + b$, can be rewritten as $y - b = mx$. I have the students state this equation in words: "The rise equals the run times the slope." Conversely, point-slope form, $y - y_1 = m(x - x_1)$ can be written as $y = m(x - x_1) + y_1$ and be said as "the y value equals the starting y plus the rise" or "the y value equals the starting y plus the product of the run and slope." Furthermore, what we call slope-intercept form is merely the case where $x_1 = 0$. Applying the same concept to quadratic functions, students are able to translate parabolas, a topic normally left to the end of the next course!

In the K-12 system, numerous pieces of research support the effectiveness of inductive instruction. The few schools sticking with CPM have enjoyed great success. Compared to the California state average, 46%

more eighth graders using CPM have scored proficient or advanced on the Algebra 1 CST over the years 2004-2010 (CPM, 2013). A study comparing CPM to traditional instruction found greater test score gains in the CPM group than the comparison (non-CPM) control group (Ferguson, 2010). At the CSU Stanislaus 2013 Math Camp, instructed in an inquiry, project-based format, students improved their test scores by an average of 26% (Sweeley, 2013). Evidence from many studies also suggests that inductive instruction fosters higher retention than traditional instruction. One particularly strong study broke 68 sixth graders into four heterogeneous classes: two with inquiry-based instruction and two controls with traditional instruction. The inquiry-based classes achieved roughly 11% higher retention, the gains increasing over time (Serrano, 2012).

Students who learn math inductively are also more likely to adapt their skills to different problems and apply concepts to different contexts (Ferguson, 2010). Driven by conceptual understanding rather than rote memorization, inductive learning surpasses direct instruction in retention and flexibility. Other research suggests that, through inductive learning, students also acquire critical thinking skills and that these broader skills can be transferred to other content areas (Bangert-Drowns & Bankert, 1990). The philosophy of inductive learning, particularly Vygotsky's brand of social constructivism, concurs with what is achieved through doing, discussing, and reciprocal teaching – not by being lectured at (Dunlosky, Rawson, Marsh, Nathan, & Willingham, 2013). Additional research has offered information about how writing improves learning of mathematical content (Bagley & Gallenberger, 1992) and tutoring improves long-term retention (Fitch & Semb, 1993).

While inquiry-based instruction arguably offers more powerful learning than direct instruction, it must be deftly executed. My classroom experience and student comments concur with Dr. Sallee's reported research on the following points (Sallee, 2003, 2013).

(1) Inductive instruction, while student-centered, does not diminish the role of the teacher. First, planning inductive instruction requires much more thought and time. Second while the teacher tells less information directly, the teacher's new focus, as the guide of conceptual architecture, is to ask questions that lead students in the right direction. Third, explorative activities require excellent directions; in this area the author has adopted more thorough and explicit instruction. Compared to direct instruction, the teacher's role in inductive instruction is lesser in providing the content but greater in providing the learning experience. In addition, the teacher's time not spent in front of the class is usually spent moving around the room, working with groups or individual students.

(2) After an exploration, it is crucial to discuss and apply the learning. The learners should describe the data they collected, their strategies, and the ideas they formed. To help the students clarify concepts, a teacher can either ask additional questions or resort to direct instruction. Application, integral to the students' learning, also serves the teacher as a vehicle of assessment.

(3) Inquiry-based instruction does not dispense with direct instruction. Some direct instruction is always needed, and the two instructional models are not mutually exclusive. Direct instruction often follows an exploration, and can serve as Plan B for students who did not attain the desired result.

In retrospect, if my introduction to the Pythagorean Theorem had been the abstruse statement, I would not have appreciated it as much, perhaps not even accepted it. Like a child proudly holding a just constructed Lego model, students take greater ownership of mathematics when they help to build it. A tribute to Vygotsky, teachers should remember that in addition to sharing content knowledge, they are helping young people to build their minds.

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