Journal of the California Mathematics Project
Volume **11**, 2022

Bringing Joy to Uninspired Teachers of Math
Touchstone Strategies, Part 1†

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Abstract. The first in a three-part series, this resource describes and illustrates two of eight Touchstone Strategies for teacher educators to use in their work with mathematics teachers. The article explores how to inspire teachers to find the joy in mathematics so they can support their students to do the same. Through a variety of tools, techniques, and helpful hints, the eight touchstone strategies in the series illustrate what high quality mathematics instruction looks like and how teachers can reframe their own thinking about mathematics to create deeper learning opportunities for their students. This piece, Part 1, introduces the collection and describes the first two touchstone strategies: *mathematical autobiography* and *"do" math.*

Introduction

Something surprising happened for me in the fall of 1973 during the very first days of my work as Assistant Director of Project Open Classroom, a progressive change project in Wayne and Pompton Lakes, New Jersey. I was hired to work with teachers in four school districts to assist with the districts' goals of changing teachers' practice to progressive, interdisciplinary teaching. Math was my focus. Midway through my very first set of workshops, I was shocked to find teams of teachers weeping. We were having fun, I thought, by using manipulative materials to engage deeply in conceptual math. Their words between the tears included, "How am I expected to teach this way if I never learned this way?" and "I've been teaching this all wrong for 10 years! What have I done to those kids?"

I learned pretty quickly that the emotions they felt were valid. Almost immediately I sensed the need to study this so that I could help them (and myself) deal with those emotions in order to have any chance of helping these teachers grow and change. I am forever grateful to those teachers and to one woman in particular named Ms. James. Ms. James repeatedly resisted change in her teaching. She both cried and argued with me. After two years, Ms. James was the teacher identified by the project as having the most changed classroom. It was her photo we put on the cover of our published book titled *Changeover*. It shows Ms. James sitting on the floor with two of her fourth-graders playing a concept teaching game about multiplication and factors using Cuisenaire rods. I reshaped my career research based on that work with Ms. James and all of her colleagues, and it helped me develop my mantra as a change agent: Embrace resistance.

† From an original report by Melnick (2018), this is the first of a three-part reprint with the permission of the author. The full report is available from the Bank Street College of Education website.

In the past 43 years of teaching teachers about math instruction at Bank Street College of Education in New York City, I have maintained a clear commitment to two goals. First, I relentlessly insist that teachers understand conceptual underpinnings of big math ideas and, second, I hope to inspire teachers as mathematical inquirers. Throughout the years, I have encountered resistance in achieving both of these goals. Therefore, as a progressive educator, I have come to expect resistance to unfamiliar ideas. What I have learned through observation, research, and study is that, for the most part, the resistance is emotion-laden.

Disciplines like mathematics, science, anthropology, or even art in and of themselves, do not carry emotions, but learners surely conjure up feelings about learning them. For many well-researched reasons, feelings about math appear to be overwhelmingly negative, especially for North Americans (Battista, 1986; Bryant, 2009; Hembree, 1990).

Explanations for why elementary teachers could be uninspired teachers of math are varied. Myths exist that some people are hard-wired as math people and some are literary – writing – reading people (Boaler, 2017). Sadly, even though those myths have been debunked, I can still walk into any school today and surprisingly find some teachers who hold onto them. After years of careful personal study, I have developed my own theory as to why. I echo the words I first heard in the 1980s by Marilyn Burns, recipient of a Bank Street Honorary Doctorate and national math educator, that one learns math only by doing mathematics. She boldly suggests that you don't learn math when you simply learn about math. You learn math when you do math by actively solving real problems with friends: grappling with confusing ideas, making models to get yourselves out of confusion, asking each other questions, and arguing or debating relative solutions. After hearing Burns say this, I began to ask myself, "Might this be the source of the problem? Maybe people never actually did any mathematics themselves, ever — and maybe they never felt inspired?"

Emotion and Math

Anyone who carries the moniker "Bank Street Faculty" should be expected to address the social, intellectual, and emotional development of the learner, child, and adult alike. As faculty we refer to emotional development as the "affective realm." Research on affect and math education tends to define affect as having three subconcepts: beliefs, attitudes, and emotions. In their Psychology of Mathematics Education research summary Hannula and colleagues (2004) cite noted researcher Douglas B. McLeod, Professor Emeritus of Math Education at San Diego State, who offers distinctions between the four sub-concepts worth considering for my work with teachers. McLeod (1992) made distinctions among these and described emotions as the most intense and least stable, beliefs as the most stable and least intense, and attitudes as somewhere in between on both dimensions. Beliefs were seen as the most cognitive, and emotions as the least so. Later, DeBellis and Goldin (1997) added a fourth element, values. Most research on affect in mathematics education has addressed one or more of these four concepts. However, according to Hannula (2004), "the theoretical foundation beneath these concepts is not quite clear" (p. 106).

Multiple lines of research conclude that emotion is probably the most fundamental concept when we discuss affect. Researchers who have studied the psychology of emotions have used different approaches, and while there is no final agreement upon what emotions are, there is agreement on certain elements. Researchers seem to agree that, first, emotions are tied to personal goals. Second, emotions also involve physiological reactions that are distinct from non-emotional cognition. Third, emotions are also seen to be functional, they have an important role in human coping and adaptation (Goldin, 2000; Lazarus, 1991; Mandler, 1989; Power & Dalgleish, 1997).

Reflective Teaching

To become a successful Bank Street-educated teacher, one needs to be skilled as a reflective professional. We ask our graduate students first to reflect on what they have read and learned in academic courses and in conference group, a course in which instructors facilitate reflective conversations about the learning that happens during students' fieldwork. During fieldwork observations, advisors aim to help aspiring teachers "reflect-on-action" by discussing observed teaching moments. This "reflect-on-action" process continues throughout the year in advisement, with the hope that the new teacher will progress to "reflect-on-action" in future teaching when no advisor or coach is there to observe. The goal is for each learning teacher to become skillfully able to "reflect-for-action" in their planning and in their curriculum design (Schön, 2016).

This metacognitive process becomes better internalized for new teachers when personal emotions (the affective dimension) have been articulated, listened to, honored, and further reflected upon. One of my process goals is for new teachers to continually reflect, so that they can affirm their feelings as purposeful in the act of learning. According to Hannula and colleagues (2004)

> Consideration of meta-affect suggests that the most important affective goals in mathematics are not to eliminate frustration or to make all mathematical activity easy and fun. Rather they are to develop meta-affect where the feelings about emotions associated with impasse or difficulty are productive. (p. 113)

Many teachers I have taught indicate intensive reflections on their emotional engagement with learning during my course.

Teachers' Emotions and the Children They Teach

Recent research indicates that math-anxious teachers can have a negative impact on their first- and second-grade students' views about math and on resulting achievement (Beilock et al., 2010). Researchers at the University of Chicago found that math-anxious women teachers have a direct impact on the girls in their classes. They measured the degree of math anxiety of a team of female first- and second-grade teachers and the math achievement of these teachers' students (boys and girls). After one full year of being in any one of the math-anxious teacher's classrooms, it was more likely that girls (not boys, however) grew to: (1) endorse the belief that boys are good at math and (2) girls are good at reading. They also found that "Indeed, by the end of the school year, girls who endorsed this stereotype had significantly worse math achievement than girls who did not and than boys overall" (Beilock et al., 2010, p. 1860). Those findings suggest that in early elementary school, where teachers are almost all female, a teacher's math anxiety can have serious consequences for girls' math achievement. This could be a contributing factor to the high numbers of women who have avoided science, technology, engineering, and mathematics (STEM) study at US colleges. Although women fill close to half of all jobs in the US economy, they hold less than 25 percent of STEM jobs (Beede et al., 2011).

These conclusions bear out both in my observations as well as my naturalistic, qualitative data collecting. It is common to see only one or two men in my Math for Teachers courses at Bank Street during most semesters. Therefore, the large percentage of students attending Math for Teachers who, year after year, describe their feelings toward math as basically negative are mostly, but not all, female. I have always felt that their lack of inspiration about math will likely influence how they will teach math. My commitment stands to do what I can to help them reorganize their emotional relationship with math, so that they can be inspired and can inspire their students, be they boys or girls!

Suggested Instructional Practices for Teaching Math to Teachers

The suggested instructional practices I refer to here grow from my life of teaching at Bank Street College of Education, as first articulated in my dissertation research (Melnick, 1992). I aimed to study the nature of change that my students repeatedly told me they experienced while in my classes. I unearthed the themes of change they were experiencing before, during, and up to five years after completing Math for Teachers with me. Four themes were found: (1) grief in a graduate-level course, (2) healing, (3) reconstructing of one's math self-identity, and (4) unpacking their personal locus of change. Key strategies that emerged from that study and that I use in my instruction will be addressed in this work, which is intended as a resource for math teacher educators.

Throughout my career, in student course evaluations and conversations with fellow faculty, I have been described as an effective math teacher educator. I believe there are factors in my teaching that cause that. Most specifically, I believe it is the conscious effort I make to address the emotional component in my teaching that gives rise to these comments. I boldly address the affective realm in my instruction. In this work, I offer teacher educators a set of suggestions to consider when planning to teach groups of teachers who may present as having been taught by being told about math rather than by doing math. I will lay out eight touchstone strategies, behaviors, perspectives, or moves that I have enacted time and again as I taught teachers (see Table 1). Each is designed to help reveal the feelings students have about math and their perceptions of themselves as math thinkers.

Anyone teaching Math for Teachers at Bank Street or a similar course elsewhere may consider some or all of my indicated touchstone strategies. I use the term touchstone since I believe the term best characterizes strategies that include the affective dimension of teaching mathematics. I offer this truncated set of ideas in the spirit with which our annual Bank Street professional appraisals are conveyed: through consistent and self-revealing generative inquiry.

Table 1. Touchstone Strategies*

Strategy	**Brief Description**
1. Math Autobiography	Use a math autobiography as an in-class assignment.
2. "Do" Math	"Wow" students in the first class meeting by doing lab-type experiential tasks across Nursery School through Grade 6.
3. Collaborative Math	Model how collaborative group work is a special kind of group work.
4. Honoring Mistakes	Model how to honor mistakes and see them as opportunities rather than failings.
5. Journaling	Monitor everyone's learning through a math journal that is linked to class readings.
6. Work a Problem to Death	Work one intentionally perplexing problem to death, unearthing confusions that arise.
7. Non-Dominant Language	Teach one class meeting in a language other than the dominant one.
8. Concept Teaching Games	Have each student plan and share their own concept teaching game.

*The first two strategies are detailed in this article, Part 1, and others are the focus of Part 2 and Part 3.

Touchstone Strategy #1: Math Autobiography

Use a math autobiography as an in-class assignment in the first meeting of class.

Start with a reflective math autobiography in the very first meeting of class before you even introduce what the course is all about. Include in the math autobiography a simple request for just one word that captures each teacher's personal memory of learning math while in elementary school.

> *Please take a few quiet moments to reflect upon your memories of your mathematical life in your elementary schooling. What feelings did you develop during those years about yourself as a mathematically thinking person? What do you remember math lessons were like? What images are conjured up as you recall your teachers teaching you math? Include specific anecdotes if you recall them. Take as much time as you need to write this.* Follow up with: *Now, write just one word that captures your personal memory of learning math while in elementary school.*

Then, ask everyone to introduce themselves to the class by telling where and what age they teach and to share the one word they wrote. Write their words immediately on the whiteboard using three columns (see a representation of this exercise below), with strongly negative terms on the left, strongly positive terms on the right, and terms that carry little or no affect in the center. I ask them to talk to their neighbors about the responses. I push them to do some math and ask, "What percentage of the people in this class had less-than-positive feelings about their memory of learning math when they were in elementary school?" Those who didn't like math might not feel so alone. Those who liked math might begin to recognize a problem they didn't know exists: "This might happen to my kids, too!" In Table 2 is the chart compiled from one recent class, categorized as I might write on the board.

Table 2. Example chart with everyone's "one word" on the first day of class.

Negative	Words with Little Emotion	Positive
work	non-existent	pleasant
nightmare	memorization	Mr. Katz (neat)
pressure cooker	vague	confident
demeaning	unexceptional	tantalizing
useless		
dry		
conflicting		
yuck!		
blah		
murky		
threatening		

With these answers, we figure four-fifths or 80 percent of this class had less than positive memories. I tell them this is my challenge and my assistant teacher's challenge for the semester: How can we influence these feelings?

The word clouds in Figure 1 depict the attitudinal shifts in my courses before and after the eight humanistic, sensitive touchstone approaches are used to instruct teachers in constructivist progressive mathematics. The power of the language used in these anonymous one word feelings by the end in the last three semesters of classes I taught should convey why I believe that these eight strategies I have used are worth sharing.

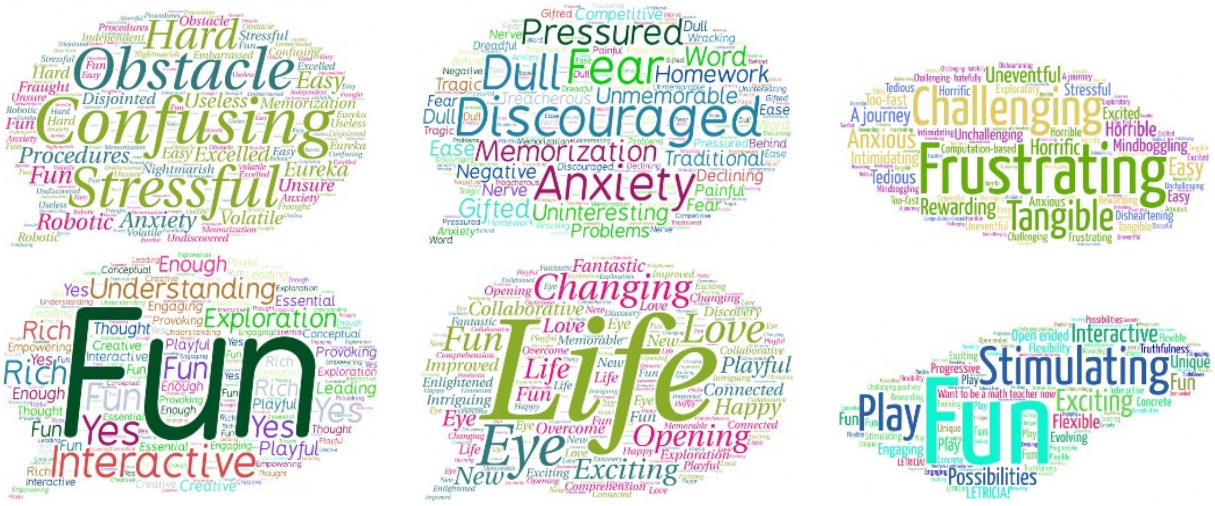
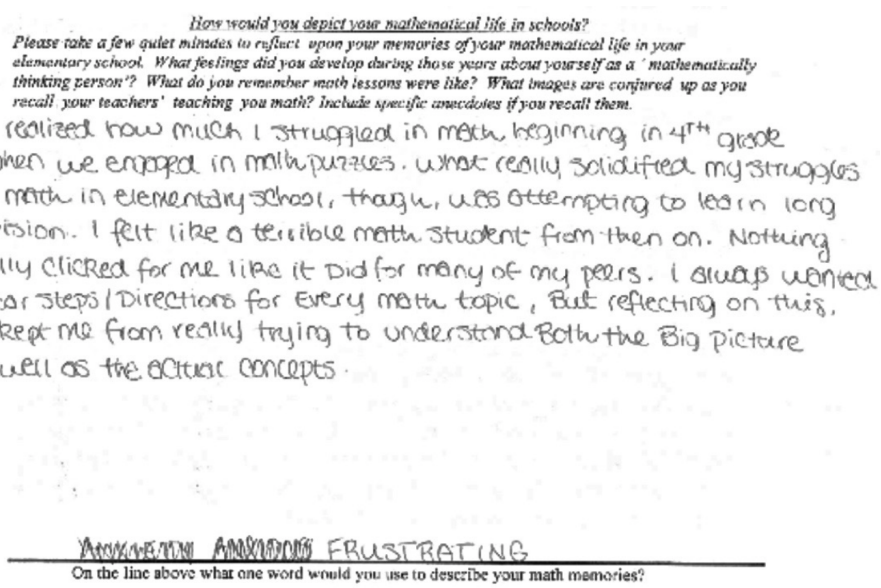


Figure 1. Word clouds for "one word" used before (top) and after (bottom) attending EDUC 540 at Bank Street. Each pair is from a different semester.

My teaching assistant and I make copies of everyone's math autobiography and read their one word and their further autobiographical comments with great interest (for a sample, see Figure 2). The first words they choose in Math for Teachers effectively help us tweak our plans for the balance of the course. We reference these words in many future activities. In private responses and individual responses to students, we often remind participants of the word they used on the first night. We ask them to continually compare their old and new feelings. I publicly admit to them that my not-so-hidden teaching agenda is that a reflective teacher will be a wiser teacher.

I have thought a lot about why these negative words keep on being selected by student after student. Based on data from math education classes I have taught for the past 43 years, prior to our first class at Bank Street, mathematics was done by only 20 percent of people attending Bank Street classes. The other


How would you depict your mathematical life in schools?

Please take a few quiet minutes to reflect upon your memories of your mathematical life in your elementary school. What feelings did you develop during those years about yourself as a 'mathematically thinking person'? What do you remember math lessons were like? What images are conjured up as you recall your teachers' teaching you math? Include specific anecdotes if you recall them.

I realized how much I struggled in math beginning in 4th grade when we engaged in math puzzles. What really solidified my struggles in math in elementary school, though, was attempting to learn long Division. I felt like a terrible math student from then on. Nothing really clicked for me like it did for many of my peers. I always wanted clear steps/directions for every math topic, But reflecting on this, it kept me from really trying to understand Both the Big picture as well as the actual concepts.

FRUSTRATING

On the line above what one word would you use to describe your math memories?

Figure 2. Sample of the Math Autobiography (Emily, 2017).

80 percent typically learned about mathematics, often in a rote fashion, and were usually taught by teachers who diligently gave them rules, formulas, and routines to memorize. Those teachers then expected their students to regurgitate what they were told, not necessarily to consider whether or not they understood any of those rules or formulas.

In addition, these students tell us that they learned not only to study for a math test and to get right answers by memorizing rules and formulas, but also not to expect to understand the reasons why any of it works. Math educators today know that a child should never be asked to memorize something he or she does not understand. The standards documents by the National Council of Teachers of Mathematics (NCTM, 1989, 1991, and 2000) and the Common Core State Standards (2010) emphasize teaching math for understanding. If math is taught simply to memorize, what may result for learners is a kind of math thinking paralysis, which is useless for building upon in later years (Boaler, 2016).

These short math autobiographies begin to signal to students my request for their feelings. I let them know I am just as interested in how they feel as in what they think. This continues in their journals, where they write about their course readings, their class experiences, and their feelings about our class. The feelings depicted in the "before" word cloud visuals are generally the result of being taught day after day with a rote approach. Since there are so many variations to the rote teaching approach in the literature (Mayer, 2002; Munter et al., 2015), I have come to call it the "teaching by telling" method. Learners in teaching by telling math classes never really had to do much other than memorize predetermined rules, formulas, and procedures that other people once discovered when they were doing math.

Here is a caveat to my above premise. Some people did just fine in math, and they were taught math with the traditional teaching by telling method. How is it that some of these adult students made it through and even went on to successfully study higher-level math and appear to now fully understand the roots of math concepts? These learners exist (although generally, I find they are only about 5 percent of those I have taught). They possess an admirable capacity to make connections between procedures and meaning on their own. Some people can connect memorized, separated facts and details, linking those disparate facts to underlying root concepts. For example, they might know that division is directly related to multiplication; in fact, the undoing of multiplication. They early on noticed that if you learned your multiplication facts, you automatically knew all your division facts. And they intuitively saw that long division was simply one procedural strategy to unearth the dimensions of a rectangular area whose area is called the dividend. They see divisor, dividend, and quotient as three numbers that are related to each other, and that all are open to multiple strategies for finding and visualizing all three as parts of a rectangle. They see fractions as division because it is! They can then use this information flexibly, like engineers do in design work or as others do in fields requiring any kind of relational thought. Math is seen by them to be inherently normal thinking. Additionally, I often find out from these students that some of that 5 percent group were taught by teachers who ran math classes that allowed kids to learn by doing, not by being told. With these progressive teachers, even years ago, students explored, conjectured, hypothesized, tested hunches, worked collaboratively, and discovered meanings in their math classes.

But what about the other 95 percent of my students? Many were unintentionally rendered disabled in math because they were not allowed to do math. They merely learned about math. The feelings they were left with are clearly depicted by the obviously negative words in the "before" word clouds. In the Math for Teachers class, the math autobiography writing on the first night establishes a firm reflective stance for the entire course. The impact of those one words on learners is described by these quotes from the student summation journals I ask my students to keep as part of the course.

On completing the math autobiography in the first class:

> Even just writing that mini-math autobiography from the outset felt like a mini-math demon exorcism, and starting on a clean slate. Not quite the word count real estate to nail my 95 theses to the Church of Math Door, but it felt a bit cathartic. I was also fascinated by the words people were asked to provide in association with our math

> experiences. I believe my word was "stress," and I don't recall any of the other words having a positive connotation. I take zero pleasure from anybody else's struggle with math, but, at the risk of sounding trite, it made me feel a little less alone in my self-imposed math exile. I know that I will have to recalibrate my entire outlook and approach to the subject, which will not happen overnight and will be an ongoing process throughout my career as a teacher, but I'm ready to go there. (Caitlin, Spring 2017, start of class)

At semester's end, I ask that you again listen to Caitlin's feelings expressed. Consider her evolution. Here she reflects upon the word she wrote in her math autobiography at the start of the course. She writes about her evolution as a learner in our Math for Teachers class. She sums up what happened to her over the semester:

> For the life of me I can't find my original math autobiography, but I luckily detailed what I had written in my first journal for this class. The word I picked when asked to sum up my experience with math was "stress." While I had blocked out a lot of my painful math experiences–many of which I've recovered from through the process of autopsying my own math education in our journals–I very clearly remember my last day of math in college and feeling a sense of elation. Pure joy, really. I was done with math and would never have to deal with math again. Flash-forward a decade and I'm undergoing a career change going from the magazine world to the education world and, surprise! Apparently math wasn't done with me. I dreaded our first math class, not knowing what to expect. But, at the risk of sounding completely cheesy, I truly feel like a different person with the subject four months later. Mainly, that I no longer fear the subject the way I did. I realize that my relationship with the subject will continue to evolve and develop through the years, more so when I actually start teaching, but I feel completely empowered. I'm actually shocked by how much, truly. I can't press the reset button for my own trials and tribulations with math, but as a teacher I now feel that I have the tools and resources to prevent my future students from experiencing what I did. (Caitlin, Spring 2017, end of class)

I firmly hold that the experience of writing a reflective math autobiography on the first night of class provides a firm starting point for growth through reflection.

Touchstone Strategy #2: "Do" Math

"Wow" students on the first night of class by doing lab-type experiential tasks for nursery school through grade 6.

With little fanfare (but a lot of preparation on my part before students arrive) I intentionally aim to "wow" my students right away. My goal is to shock them with a colorful and joyful view of what math teaching can look like across grades N-6. Graphs with materials to manipulate, and at times even taste, are up on walls or on tables for different stages of abstraction. A classic racing dice game is there to be played in Kindergarten or in more depth at Grade Six. Charts exist on walls with ponderable math questions, materials are on the table for measuring and drawing conclusions from those measurements about the idea of the mean, etc. These tasks surround the room at lab tables and on walls (Figure 3 has photos of six examples).

As soon as they fill out their math autobiography, they get up and do the 11 or 12 tasks around the room. I always tell them to find a friend and work cooperatively on the tasks for support.

They wander with smiles, happily tasting white or purple grape juice and place their empty cup on one of two columns on the vinyl coated graph. The resulting lines of cups demonstrate a three-dimensional "real"

graph showing drink preferences for 4- to 5-year-olds. A noticeable active noise erupts and laughter ensues. They are immediately "doing" math. At one learning station, they are expected to measure their heights with little white Cuisenaire Rods, graph the data using post-it notes, and ponder the shape of the emerging graph and where the next person will most likely place their post-it. At another station they have to toss two dice as many times as they can, recording on a 2-12 chart when each sum turns up. Which sum do they anticipate will win? Why? A question on their worksheet asks which sum they think will reach the top of the grid first. They enter their results and we compile a Sums that Won Class Chart. What is the theoretical probability of your "winning number" winning the next time you play? This and about eleven other tasks across the age span generate a lot of open conversation, laughter, and pondering about what the course might entail.

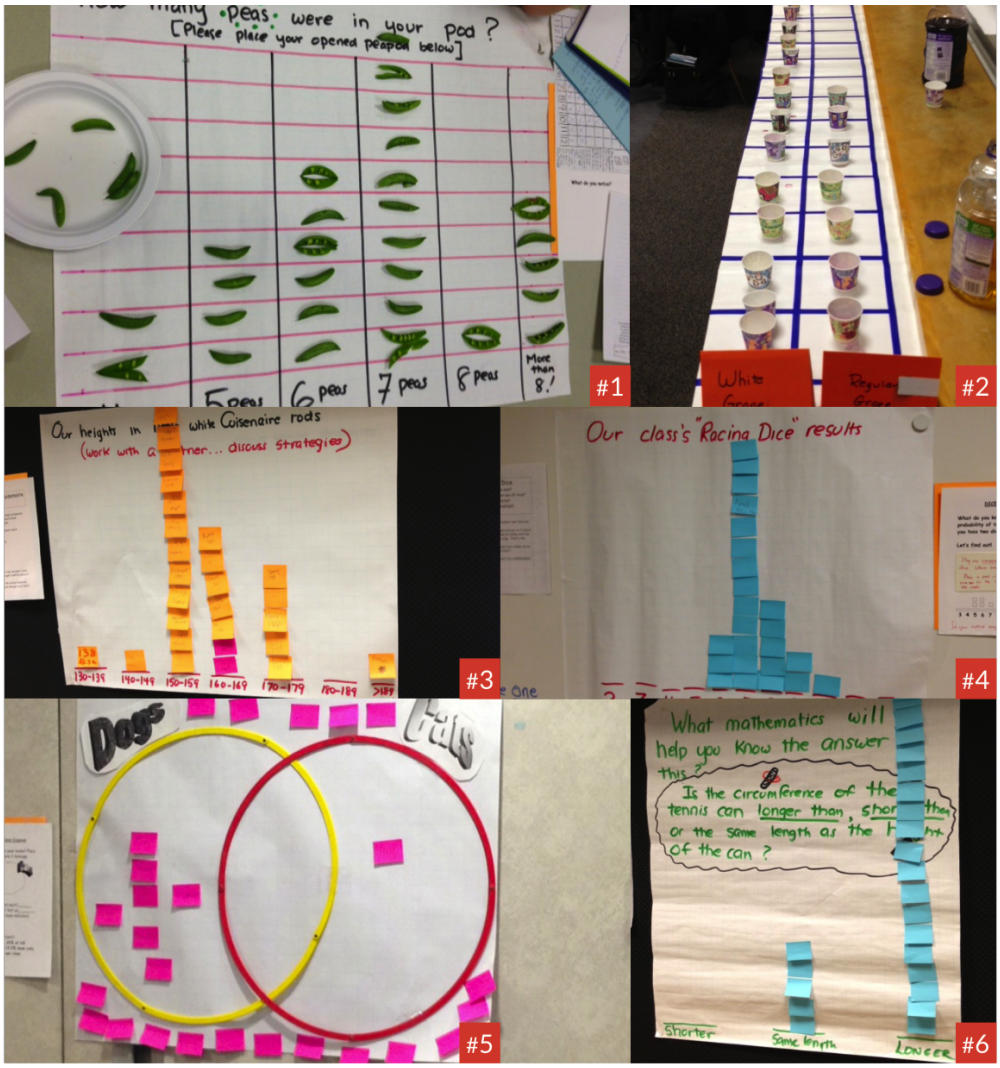


Figure 3. Six examples of tasks done by students in the first class meeting.

#1 How many peas are there in your pea pod?
#2 Our preferred juice drink.
#3 Our class's heights in white Cuisenaire Rods.
#4 Our class's Racing Dice results.
#5 Do you have a dog, a cat, or neither at home?
#6 Is the circumference of the tennis can longer than, shorter than, or the same length as the tennis can itself? What math that you likely learned in middle school will help you answer?

Things they write inform me about their reaction to that night and invariably begin to inform my assistant and me about their prior math lives. Right away we start hearing expressed emotions, fears, excitement, and expressions of attitudes and values. The affective realm is out there in full view. The seeds for the course are planted.

Here are a few of their comments evidenced in their math journals after the very first night of class:

> Doing the exercises, from the racing dice to the pea pod estimations, was really compelling. Something that might seem simple on the outset actually stimulates much deeper thinking, even in adults. Why did most people end up with sevens and eights as a total with the dice? Why did people choose the white grape juice over the red? These group activities made me realize that I will have to be in a constant state of questioning and will have to likely break up any preconceived notions and assumptions I've had in the past, in order to get the most out of this course (Caitlin, Spring 2017).
>
> Last week we had our first class of Educ 540. Walking out of class that night, I felt something I had never felt before; excited about math. To me, this was a big deal. The reason being that in the past, whenever walking in or out of a classroom where the topic involved math, I would have been overcome with anxiety.
>
> This class began with our professor asking us to recall our experience with math before grade six and one overall word to describe this experience. I wrote down some of the phrases my classmates shared; stressed, confusion, anxiety, trauma, and horror. Most of my classmates shared a word that implied their experience with math before grade six was negative. I began to analyze why exactly my experience was so negative, and I realized that learning math lowered my self-esteem and confidence. I was unable to grasp the concepts fully we were learning in math class, and this caused me to classify myself as "not smart." This feeling followed me each year, and before I even entered into a new math class, I already had that feeling of inevitable failure.
>
> However, this class made me excited about math because I want to learn how to best teach math in a way that students will not feel the same way myself and my classmates felt. There are so many fantastic resources and ideas about teaching math in a way that fully supports students, and I can't wait to learn about them. My professor also made it clear that he believes each student learns best in a different way. This philosophy made me hopeful as I wholeheartedly agree and believe in the importance of considering each student individually. When I was younger, I wished my teachers would have considered the fact that each student learns differently and just because one strategy worked for my classmate, that didn't necessarily mean it was appropriate for me (Katherine, Spring 2017).

I am always amused by the delight on people's faces during this first session. I observe and take note of each person's reaction. Is there a sign of joy; of fear, of confusion? Does anyone appear very uncomfortable? Towards the end of that first class I remind people of the electronic link that was sent to them a month earlier in my introductory welcome letter. The emailed letter had included the course outline, lists of assignments, and required books for the course, however I never spend the first night with such details. I ask them to carefully read those materials again and come with questions, all of which will be addressed next session. My intention is to be invitational and inspirational on the first night. I will admit that occasionally some people drop the class as a result. Sometimes they are just not ready to handle the kind of shift this night lays a foundation for. When they are ready they will return.

Acknowledgements

With deep appreciation to all my former students whose shared insights have taught me how to teach them. This report was supported by the Learning Agenda at Bank Street College of Education with generous funding from the Carnegie Corporation of New York. It was authored by Dr. Hal Melnick with vital assistance by Jessica Charles. Mona Rayachoti and Jonayah Marie Jackson led the graphic design for the report.

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