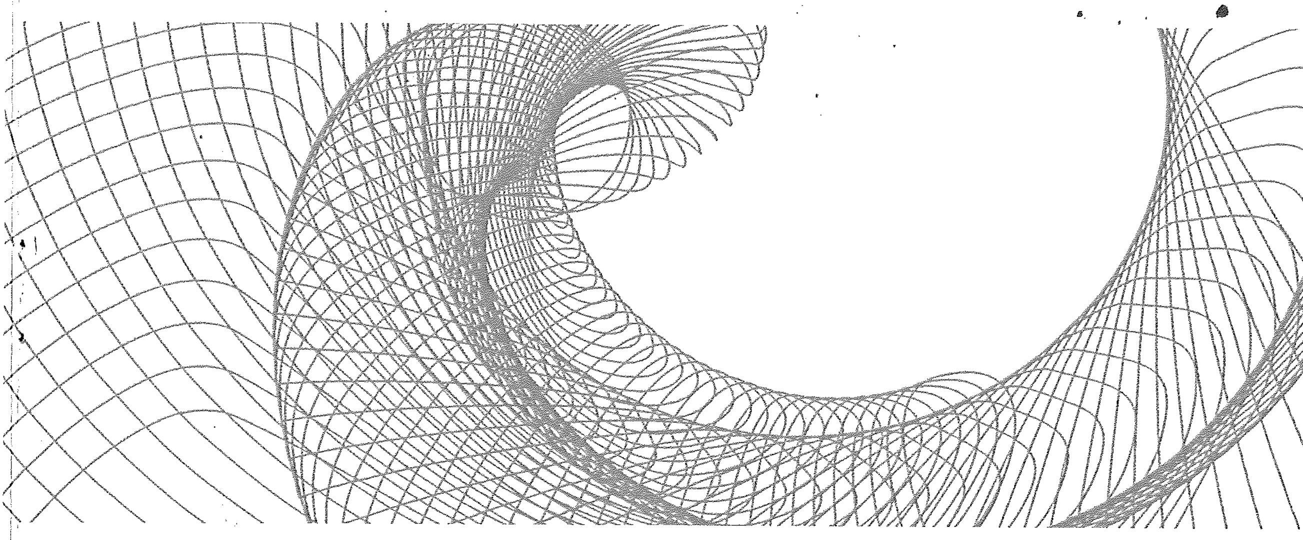
journal of the central california mathematics project



California State University | Stanislaus

table of contents

Introduction

by Suzie W. Hakansson, Ph.D.

The mission of the California Mathematics Project (CMP) is to enhance K-12 teachers' content knowledge and instructional strategies aligned with the *California Mathematics Content Standards* and *Framework* through the collaboration between mathematicians, mathematics educators, and teacher leaders.

The Central California Mathematics Project (CCMP) at CSU Stanislaus, one of 19 CMP sites, has been a CMP site since 1984. Throughout the 24 years, CCMP has served many teachers and students, providing a variety of professional development programs to increase teachers' effectiveness in the classroom. CCMP has provided programs for schools and districts including teachers of English learners, teachers seeking subject matter competency, and teachers in high need schools and districts. During many of their summer programs, teachers from Thailand were invited to participate. This partnership with Thailand has broadened the perspectives of all teachers participating.

This inaugural CCMP journal is the result of years of effort, discovering what works for teachers. CCMP is sharing what has worked for them. To prepare for the writing of this journal, CCMP held a summer institute during 2008 for participants interested in writing articles for this journal. Consultants with expertise in mathematics and/or writing supported the participants, reading many drafts and providing technical assistance. I had the privilege of visiting CCMP during this institute and fit right in since I had a writing task to complete that week. It was great to see the participants so engaged in their writing efforts, so much that they were often oblivious to their surroundings.

From elementary classroom activities to overviews of university coursework, this journal has something for all mathematics educators and teachers of all grade levels. It is a credit to Dr. Viji Sundar, Director and Faculty Advisory of CCMP, for her vision to accomplish this work.

Enjoy reading and using the activities.

Susie W. Hakansson, Ph.D.
Executive Director
California Mathematics Project

Foreword

by William Covino

One of the primary goals of the Central California Mathematics Project is to "Develop instructional strategies to improve academic performance of students in mathematics." This collection contributes a great deal to this goal, while it also reinforces and exemplifies the commitment to excellence in teaching at California State University, Stanislaus and throughout the six-county region that the CCMP serves.

As an internal publication of the CCMP, this collection features math lessons and best practices that have been classroom tested, and which will surely empower both teachers and students. With the assistance of Professor Viji Sundar, CSU Stanislaus student Deidre Rodriguez provides an engaging approach to drawing the dimensions of sports fields; La Loma Junior High School math teacher Mary Nay gets at fundamental math concepts through popular comic strips; Ceres High School math teacher Ramina Isaac helps students see that math is a key to understanding real-world problems; Caswell Elementary School teacher Mary Gonzales provides an amusing and memorable poem that helps third-graders learn to round numbers; and Merced College math instructor Stephanie Souza points us toward career skills, with an intensive math course developed for Forest Service employees. In addition, CSU Stanislaus Professor Heather Coughlin presents both an essay on the challenges elementary school teachers face when creating word problems that involve dividing fractions, and an engaging class activity that involves students in calorie-counting and price comparisons. Further representing the breadth of interest and participation in this project, Professor Ruangporn Prasitkusol from Phranakhon Rajabhat University offers a model university course in "Statistics for Research; Dr. Wimol Sanguanwong, from Wat Pra Srimahadhat Secondary Demonstration School in Bangkok presents approaches to teaching quartiles to high school students, and Viji Sundar and Marie Vanisko (from Carroll College in Helena, Montana) present a course in linear and abstract algebra for non-math majors.

The range of topics in this collection is impressive, and is designed to appeal to a number of teachers in our region. In addition, it exemplifies collaborative energy, provides a valuable look at effective approaches to pedagogy and curricular development, and re-emphasizes the dedication to student success that we all cherish. I am very grateful to Professor Sundar and her colleagues for putting together this collection, and trust that you will all read and profit from its lessons and advice.

William A. Covino

Provost and Vice President for Academic Affairs
California State University, Stanislaus

Geometry in Athletics
Learning the Dimensions of Sports Fields

DEIDRA RODRIGUEZ, *Student, CSU Stanislaus*
VIJI SUNDAR, *Professor of Mathematics, CSU Stanislaus*

Strands:
Number Sense; Measurements and Geometry.

Skills:
Ability measure and draw football field and soccer field on a standard piece of paper (8½"×11") or graph paper (for lower primary grades) using geometric tools. Learning the correct vocabulary for the lines, rectangles, circles, arcs that appear on the two fields.

Mathematics Standards:
Gr. 3: MG- 1.1, 1.4, 2.3, 2.4
Gr. 4: MG- 1.2–1.4. 3.1, 3.2, 3.5
Gr. 5: MG- 1.1, 1.4, 2.1, 2.2
Gr.6: AF- 2.1, 3.0. M/G- 1.1, 1.2
Gr. 7: MG- 1.2, 2.1–2.4, 3.1

Grades:
3–7

Materials:
For each student two sheets of Standard 8½"×11" paper and/or graph paper, a 12" ruler, a protractor, a compass, and a calculator. Also pencils, and one copy each of the football field and regulation soccer field. (Attached)

Background:
Math is intertwined with everyday activities that we take part in. Students and even athletes may not be aware that the field in which they play is full of mathematical shapes and their game rules are based on the shape and size of the playing field. Whether it is baseball, football, soccer or basketball, the field is scattered with many geometric shapes and angles that are found in elementary classrooms and textbooks. Research shows that teaching math to students is easier and more exciting if the concepts relate to their experiences. This article will explore many activities for students to compare a football field with that of a soccer field. The lesson can be a forum to learn precise vocabulary and use geometric tools for precise construction.

The writing of this article was supported by the Central California Mathematics Project.

Even some of the ardent fans of the games may not know that the dimensions of football fields are constant irrespective of the age and ability of the players. In soccer, the dimensions of the field will vary depending on the ability of the team and the competitive level of the players. As a starting point to this activity, teachers may begin a dialogue with the students about their observations of the playing fields, game rules, scoring and more.

The activities here can be modified for different grades (3–7) because the skills required and questions asked may be varied to meet grade level standards. At every grade level, the lesson should begin and end with a 'walk through' of the football and soccer ball fields.

Description:

The purpose of this article is to review and re-teach some geometric concepts, vocabulary and construction in an engaging and fun way. The students construct their own football and soccer fields with pens, colored pencils, rulers, compasses, and graph paper. That is to say, the object is to draw a scaled down version of the fields on a 8½" by 11" (graph) paper and compare the two fields for shape, size, area, and perimeter. As an extension one can discuss the relationship between the rules of the game and their respective fields.

Directions:

1. The teacher and the students discuss the differences and similarities between the two fields. This is also a great time to introduce geometric shapes in primary grades and maybe review them for the upper the grades. The teacher may discuss with the students about why there are different size fields for the two games.
2. Assemble all of the materials needed for the project and provide them to every student.
3. Hand out attached worksheets with diagrams of a soccer field and a football field on it. The dimensions are shown in yards.

Activity I: Drawing the Football Field

1. The students will first draw the boundary lines of the field using an appropriate scale.
2. The students will then section off the areas designated for the end zones.
3. The students can then move on to placing the yard lines on the field, making sure that the 50 yard line is in the center and moving from 40 to 30 to 20 to 10 to endline on either side. Mark off as shown in the figure.
4. After drawing the ten-yard lines the students will put the hash lines as shown on the diagram.

Activity II: Drawing the Soccer field

1. Students will draw the soccer field as indicated in the figure given.
2. The students will then draw the midfield line. (Midfield line is the perpendicular bisector of the sidelines)
3. Next, the students draw the center circle as shown in the figure.
4. The students draw the two goalie boxes, located on the outside of the perimeter of the field.
5. Draw in the Six-yard box, which is the smaller of the two boxes on either end of the field.
6. Draw the 18 yard box which is the bigger box surrounding the penalty box.
7. Draw the penalty spot, which is shown as the half way point between the top of the penalty box and the top of the 18 yard box.
8. In the final step, students will draw the top arch on the top of the 18 yard box. This arch has its center at the penalty spot and the radius is the same as the center circle.

Grade appropriate project and curriculum based Ideas: **Note that any of these activities can be used for grade levels other than where they are seen.**

THIRD GRADERS:

- Have the dimensions of the fields be shown in feet, and then have the students convert them to inches. This allows the students to learn simple conversions.
- Have the students study the diagram; have them tell you where they see parallel and perpendicular lines.
- Also, have them study it to see what angles they can recognize by producing the parallel and perpendicular angles.
- The teacher can also have the students count the number of rectangles and squares they can find in the diagrams.

FOURTH GRADERS:

- Have the students find the perimeter of the football and soccer field.
- Have them study all of the rectangles noticing the differences between the sizes of the rectangles shown. They can use their colored pencils to highlight the different rectangles and squares.

- Have them study the fields to find the different degrees produced in each diagram. They should be able to find 90°, 180°, and 360° angles.
- The teacher can introduce or review arc, the radius and diameter of a circle. (Using the soccer field)

FIFTH GRADERS:

Now the students should be able to reproduce the football and soccer fields without looking at the diagrams; they can create the field with just the dimensions given.

- Teachers can take their students outside to the actual fields and have them measure it to see how they relate to diagrams that they drew in class.

SIXTH GRADERS:

- In addition to the above exercises students can find the circumference and area of the circle in the soccer diagram.
- The students can measure the fields, making it a little more challenging. This will allow the students to practice converting and bring the fields down to scale.

SEVENTH GRADERS:

- Students here should be able to reproduce each field given actual dimensions. (Yards, meter, centimeters)
- They should also be able to recognize the difference in units, making sure that their conversions are appropriate. (Ex. centimeters to inches, meters to inches.)
- Students will explain how and why they used the units in their diagrams.
- Students can make their fields three dimensional. Showing goal posts, penalty boxes, etc., using cardboard or another thick type of material. The students can then find the surface area of the walls.
- Using the soccer field diagram the students can find the diameters, radii and chords.

VOCABULARY WORDS

Football Field Vocabulary: Refer to the Directions Section under Football Field

Sidelines	The longer lines that make up the boundary of the rectangular field.
Touchlines	The boundary lines that mark each sideline of the field
End Line	The shorter lines that form the boundary of the rectangular field.
End Zone	The two end zones, one for each team, represented in the diagrams as the last 10 yards at the end of the field. This is also where the teams score touchdowns.
Goal Line:	This line forms the front portion of the end zone; the back end of the end zone is the end line. It would be the zero yard line on either end of the field.
Yard Lines	These are the lines that divide up the field evenly every 5 yards.
Hash Marks	The marks inside of each yard line; each hash mark represents 1 yard.
Yard Number	The numbers that are drawn on the field. (10, 20, 30, 40, 50)

Soccer Vocabulary: Refer to the Directions Section under Soccer Field

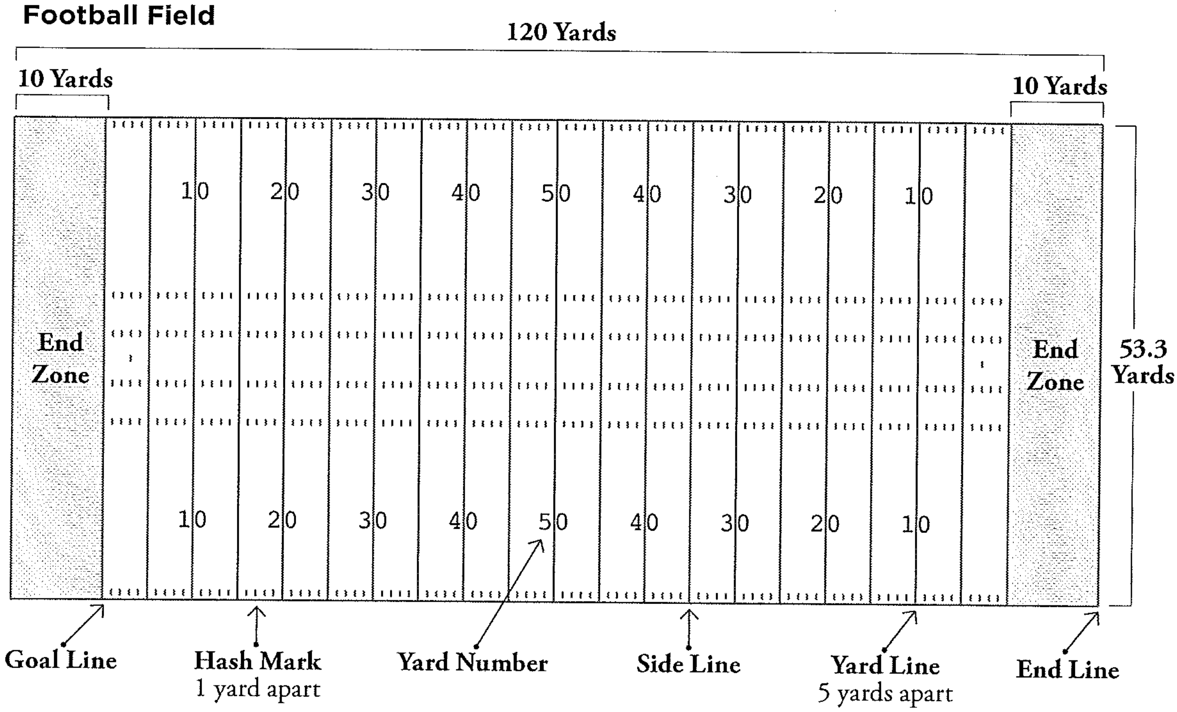
Touchlines	These are the lines that form the boundary of the field.
End Lines	The shorter lines that form the boundary of the rectangular field where the goal boxes are.
Halfway Line	This is the line that divides the field equally into two separate sections.
Center Circle	This is where the game starts; the team that has possession of the ball is allowed to stay inside the circle to start. The opposing team has to remain outside the circle. Center Circle is the starting point after scoring a goal. The radius of center circle is 10 yards.
Center Spot	The geometric Center of the center circle where the ball is placed at the beginning of each half of the game and after a goal is scored.
Penalty Box or the 18- yard box	This is where penalty shots are awarded for each foul. The dimensions of the penalty box are 44 yards by 18 yards.
Penalty Spot	This is where a shot on the goal takes place if a player is fouled.
Penalty Arc	This is an arc that extends outside the penalty area, marking 10 yards from the penalty spot.
Six Yard Box	The rectangular area from where the goalie may pick up the ball and kick it or throw it to a teammate.
Goal Box	This is the touchdown place for scoring a goal and the home of the goalie

References:

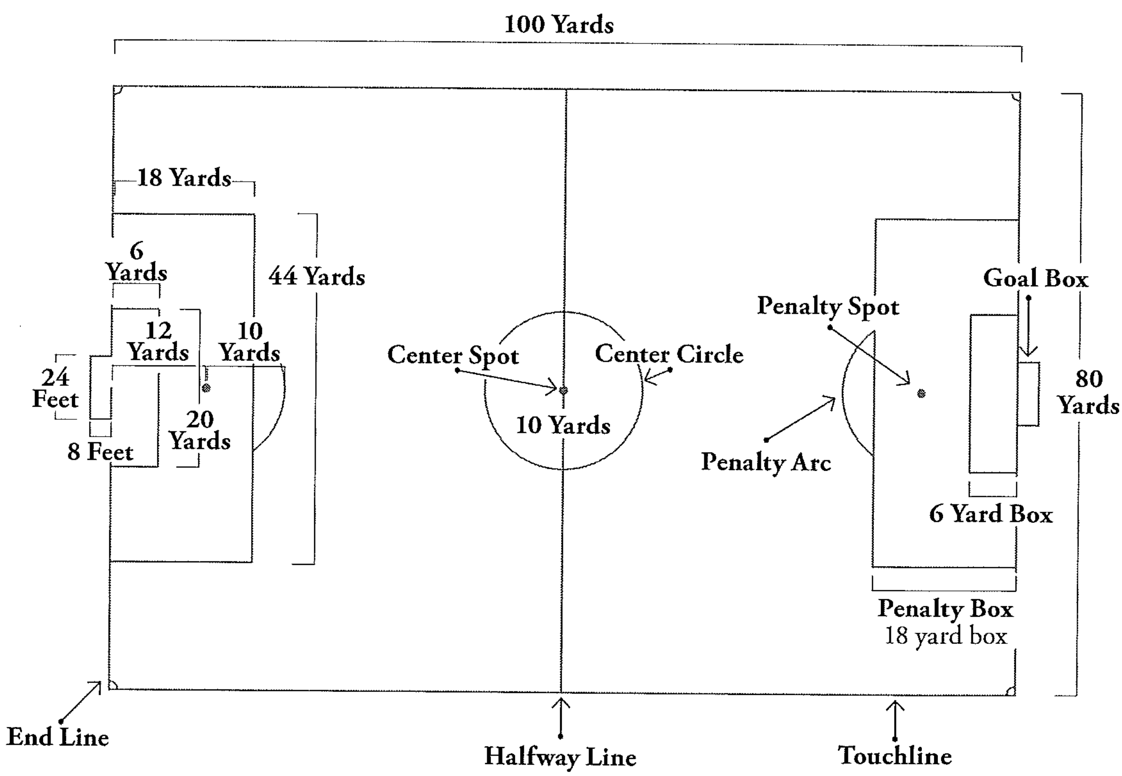
Football Vocabulary from: http://football.about.com/od/football/101/l/bl_glossary.htm

Soccer terms from: http://ezinearticles.com/?A-Glossary-of -Soccer-Terms,-Definitions,-and-Terminology&id=457668

Football Field



Soccer Field



Cartoon Corner!

MARY NAY *Math Teacher, La Loma Junior High*
VIJI SUNDAR *Professor of Mathematics, CSU Stanislaus*

"Cartoon Corner" is a regular feature in the NCTM journal "Mathematics Teaching In the Middle School". The inclusion of this in the journal, to quote from the journal, " is to highlight mathematics in an interesting way. The questions accompanying the cartoons can be used as a large or small-group activity, as homework, or as an extra credit assignment."

Article authored by:

Mary Nay, Math Teacher, La Loma Junior High School and

Viji Sundar, Professor of Mathematics. CSU Stanislaus

The writing of this article was supported by the Central California Mathematics Project.

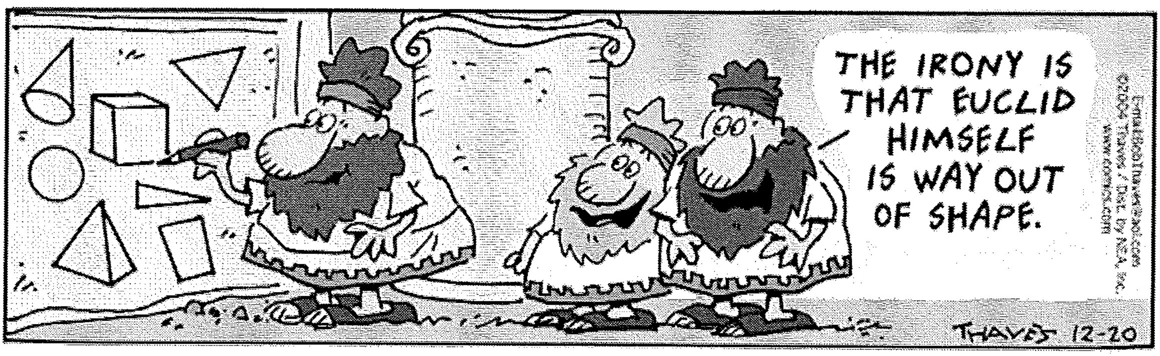
FRANK AND ERNEST Reference: http://frankandernest.com/cgi/view/display.pl?100-04-07

1. a) What does "multiply" mean in the cartoon?
   b) What does "multiply" mean in math? Explain with an example.
2. a) How is multiplication related to addition?
   b) How is division related to subtraction?
   c) Explain multiplication as addition and division as subtraction with examples.
3. a) What is 5 times 5 times 5 times 5?
   b) How can you rewrite this operation?
   c) What is this operation of repeated multiplication called?
4. Does 20 – 5*3 equal 45 or 5? Explain why.
5. a) What is the rule of operations?
   b) Why do we have rules for operating in math?

MATH CLASS Reference: http://frankandernest.com/cgi/view/display.pl?108-01-03

1. What is meant by the comment above?
2. What does "x" mean in Algebra? Give some examples.
3. Find "x" given .
4. a) If you start of with $20 and have $8 left after spending "x" dollars, write an equation using "x".
   b) Find out how much you spent by solving for "x".
5. Write a math problem of your own using "x", then give it to a friend to solve.

SHAPE Reference: http://frankandernest.com/cgi/view/display.pl?104-12-20



1. a) Explain the context in which the word "shape" is used in this cartoon.
   b) What does the word "shape" mean in geometry?
2. Find the number of sides of each of the following shapes:
   a) Triangle b) Octagon. c) Pentagon. d) Hexagon. e) Square. f) Heptagon.
3. Find the area of a square park 250 yards long.
4. a) Name all the shapes in this cartoon.
   b) Which of those shapes have volume and why?
5. a) Find the surface area of a cube with length 3 inches.
   b) Find the volume of the same cube.
6. Find the perimeter of a regular heptagon with sides of length 8 centimeters.
7. If the perimeter of a regular hexagon is 98 meters, find the length of each side.

TUNA

Reference: http://www.comics.com/comics/soup2nutz/archive/soup2nutz-20080627.html

1. Why did the character answer "Tuna?"
2. a) If a can of tuna can make 2 sandwiches, how many cans are needed to make 10 sandwiches?

   b) How many cans are needed to make enough sandwiches to feed your whole class?
3. a) A recipe for tuna fish casserole calls for 3 cans of tuna to feed 5 people. How many cans do you need to feed 15 friends?

   b) How many people can you feed if you only have 7 cans?

   c) Do you have enough tuna fish casserole to feed your class if you have 16 cans?
4. Find the surface area of a 3 oz. tuna can in

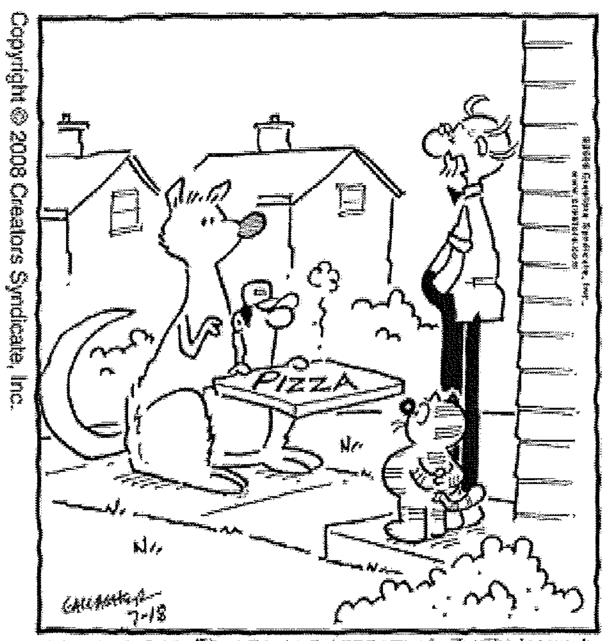
   a) Square centimeters.

   b) Square inches.
5. Find the volume of a 3 oz. tuna can in

   a) Cubic centimeters.

   b) Cubic inches.

PIZZA



"THESE GAS PRICES ARE AFFECTING EVERYONE."

Reference: http://www.arcamax.com/heathcliff/s-377412-931710

MATH IS EASY

Reference: http://www.arcamax.com/rugrats/s-378883-976264

1. According to the cartoon
   a) What is the answer to 3 plus 3?
   b) Make up the next 5 boxes in the cartoon.
   c) Explain the girl's answer.
2. Complete this pattern in 3 different ways: 1, 2, ___ , ___ , ___ , ___ .
3. a) Find the next 3 numbers in this pattern: 2, 9, 23, 51, ...
   b) Explain the pattern here.
4. a) Complete this pattern: 2, ___ , ___ , ___ , 34. Explain your answer.
   b) Complete this pattern: O, T, T, F, F, S, ___ , ___ , ___ , ___ .
   Explain your answer.
5. Create your own pattern, then give it to a friend to complete.

1. Why do you think the pizza delivery guy is delivering pizza sitting in a kangaroo pouch?
2. a) How many miles is it from your house to your favorite pizza restaurant?
   b) A car can go 28 miles per gallon and gas costs $4.30 per gallon.

How much does it cost for the pizza guy to deliver pizza to your house?

3. Use the information in problem 2. If the delivery guy has 20 deliveries in a day totaling 100 miles in travel, how much does it cost him in gas to make those deliveries?
4. Assume that your home is 3 miles from the school, your car gets 32 miles per gallon, and gas costs $4.20.
   a) How much gas does it take to go to school and back each day?
   b) What is the cost of gas for this daily trip?
   c) What is the cost for the week?
   d) What is the cost for the month? Assume 1 month = 4 weeks.
   e) What is the cost for the school year? Assume 1 school year = 9 months.

SOLUTIONS

FRANK AND ERNEST

1. a) In the cartoon, "multiply" means to increase.

   b) In math, "multiply" means the same number added multiple times.

2. a) Multiplication is repeated addition of the same number.

   b) Division is repeated subtraction of the same number.

   c) Examples: and 5 appears 4 times, so

3. a) 625.

   b) 54.

   c) This operation is called "exponentiation".

4. . You must multiply before you subtract.

5. a) The rule of operations is work inside the parenthesis first, exponents next, then multiplication or division (which ever comes first), and addition or subtraction (which ever comes first). A useful acronym is PEMDAS: Please Excuse My Dear Aunt Sally.

   b) We have rules for operating in math so that we will have the same answers.

MATH CLASS

1. Answers vary. Sample answer: The speaker does not understand the role of "x" in algebra.

2. In Algebra, "x" is a variable that represents an unknown. For example: "x" could represent the temprature outside or the time it takes to travel a certain distance or the amount of money you have in your pocket. "x" can also be a word.

   "x" is a day of the week and therefore has 7 possible answers (Sunday, Monday, ... , Saturday). Another example: "x" is a day of the week beginning with the letter T and this has 2 possibilities (Tuesday and Thursday)

4. a) $\$20 - x = \$8.$

   b) If you had \$20 and spent \$12 you will have \$8 left.

5. Answers vary. Sample answer: You had 5 pairs of shoes and got some more for your birthday, now you have 12 pairs of shoes. How many pairs of shoes did you get for your birthday? 12 pairs of shoes.

SHAPES

1. a) In the cartoon, the word "shape" refers to physical fitness.

   b) In geometry the word "shape" is used to represent a picture, figure or object

2. a) 3 b) 8 c) 5 d) 6 e) 4 f) 7

3. Area of a square = .yd2.

4. a) Triangle, square, circle, quadrilateral, cone, triangular pyramid, cube.

   b) Cone, pyramid, and cube have volume because they are 3-dimensional shapes.

5. a) Surface area of a cube = 6s2.

   = 6(3 inches)2

   = 6(9 inches)

   = 54 in2.

   b) Volume of a cube = s3.

   = (3 inches)3

   = 27 in3.

6. Perimeter = , where s = the number of sides and l = length of side. Perimeter of a heptagon: 7(8 cm) = 56 cm.

7. Perimeter = , where s = the number of sides and l = length of side. Perimeter of a hexagon: m

TUNA

1. Yes. Bumble Bee is a company that makes tuna and salmon. http://www.bumblebee.com/About/
2. a) . 1(10) = 2x, cans.

   b) Answers vary. Replace x with the number of students in your class.
3. a) 3(15) = 5x, cans.

   b) 3x = 5(7), .6 people. Since 0.6 of a person is not possible, the correct answer is 11 people.

   c) Answers vary. Replace x with the number of students in your class in 3b.
4. Surface area of a cylinder = , where r = radius and h = height.

   a) Answers vary. A tuna can is approximately 2.5 centimeter in height and 7.5 centimeter in diameter. SA = cm2.

   b) Answers vary. A tuna can is approximately 1 inch in height and 3 inches in diameter. SA = in2.
5. Volume of a cylinder = , where r = radius and h = height.

   a) Answers vary. V = cm3.

   b) Answers vary. V = in3

PIZZA

1. Answers vary. Sample answer: The pizza guy is delivering pizza sitting in a kangaroo pouch because it is too expensive to drive.
2. a) Answers vary. Sample answer: 3 miles.

   b) $\frac{\$4.0}{8 \text{ mpg}} = \$0.5$ per mile. \$0.15(3 miles) = \$0.45.
3. \$0.15(100 miles) = \$15.
4. a) $\frac{6\,miles}{3\ mpg} = 0.9$ gallon.

   b) 0.19(\$4.20) = \$0.80 each day.

   c) \$0.80(5 days) = \$4.00 for the week.

   d) \$4.00(4 weeks) = \$16.00 for the month.

   e) \$16.00(9 months) = \$144.00 for the school year.

MATH IS EASY

1. a) Threety – three.

   b) Four plus four is fourty-four, five plus five is fivety-five, six plus six is sixty-six, seven plus seven is seventy-seven, eight plus eight is eighty-eight.

   c) Answers vary. Sample answer: Instead of adding 2 and 2 she is placing them next to each other: 2+2 is 22, 3+3 is 33, etc.
2. Answers may vary. One possibility:

   1st way: 1, 2, 3, 4, 5, 6.

   2nd way: 1, 2, 1, 2, 1, 2.

   3rd way: 1, 2, 4, 8, 16, 32.
3. a) 2, 9, 23, 51, 107, 219, 443.

   b) Multiply by 2 and add 5 to each number to get the next number. Answers may vary
4. a) 2, 10, 18, 26, 34. Add 8 to each number to get the next number.

   b) O, T, T, F, F, S, S, E, N, T.

   Pattern comes from the first letter of the counting numbers.

   One, Two, Three, Four, Five, Six, Seven, Eight, Nine,Ten.
5. Answers vary. Sample answers:

   3, 6, 9, 12 , 15 , 18 , 21 , …

   5, 25, 125, 625 , 3125 , ...

Math! A World of Discovery

RAMINA ISSAC, *Math Teacher, Ceres Unified School District*
VIJI SUNDAR, *Professor of Mathematics, CSU Stanislaus*

Strands:
Mathematical Reasoning, Number Sense

Skils:
Use the Internet to research and analyze data.

Grades:
5–7

Materials:
Worksheet for students (to work in pairs), Pencil, Calculator, Computer with Internet connection and a World map (optional)

Background
Many students fear math and even a greater number fear doing 'word problems' in math. However real world problems can help students to look past fears and open minds to mathematics. It can even give them joy to successfully tackle math problems. Students should realize mathematics is used to understand many situations. People should be aware that math is part of their daily lives; there is a story behind each number; there is a reason for a number. Whether it is the price of a hamburger, the daily temperature, the number of students in a classroom, one's Social Security number, a car's license numbers, or the map coordinates of a city, connecting math to real life examples peak their curiosity.

Purpose
The purpose of this lesson is to create awareness of the relative cost of food in other parts of the world. Food expenditures differ throughout the world. The lesson is also an opportunity for students to learn how to research using the web, solve mathematical problems, become familiar with world geography and see the world with a different eye. Information and activity sheets for the students are included.

REFERENCES

Peter Menzel, Failth D'Aluisio "The Hungry Planet" Ten Speed Press: September 2007

Map retrieved on July 24, 2008 http://www.worldatlas.com

"What the world eats Part I" retrieved on July 24, 2008 from http://www.time.com/time/photogallery/0,29307,1626519,00.html

Cuernavaca, Baja California, Mexico retrieved on July 24, 2008

http://www.traveljournals.net/explore/mexico/map/m5021794/cuernavaca.html

Map retrieved on July 24, 2008 from http://english.freemap.jp/

Description

This article is interdisciplinary and is appropriate for 5-7 graders. It touches on world geography, culture and family, mathematics, research skills, and awareness of food waste around the world. The lesson is designed to make an impact on their thinking and make students aware of the world around them. The information gathered here is from the book *Hungry Planet: What the world eats?* by Peter Menzel, Failth D'Aluisio and Marion Nestle. The authors of this book set out to see how families in 24 countries in the world feed themselves each week. We have chosen 12 of the 24 countries for this lesson.

Walk the reader through the activity sheets.

Extension

At the end of the lesson, the students can determine their family food expenditures and their personal food expenditures for one week, record the information, and present the results to the classroom. (Information needed is cost and type of food consumed). This can be developed in several ways - compare each student's consumption to family food expenditure individually and as a group. This information can be charted with the names of students, their weekly food expenditures and their family size. The students may be divided into groups of 3 or 4 to discuss which family food expenditures within their group are the same or different. On completing the activity sheets, the teacher can direct the discussion to the type of food purchases and their nutritional values.

Information Sheets

The following facts have been gathered from the book called *The Hungry Planet.* It gives information on food expenditures each week for the families with the number of members in each of the family.

1. In Cairo, Egypt, the Ahmed family of 12 spends $68.53.
2. In Breidjing Camp, Chad, the Aboubakar family of 6 spends $1.23.
3. In Kodairea, Japan, the Ukit family of 4 spends $317.25.
4. In Shinkgkhey Village, Bhutan, the Namgay family of 13 spends $5.03.
5. In Kuwait city, Kuwait, the Al Haggan family of 8 spends $221.45.
6. In Bargteheide Germany, the Melander family of 4 spends $500.07.
7. In Sicily, Italy, the Manzo family of 5 spends $260.11.
8. In Collin Bourne Ducis, UK, the Baytons family of 5 spends $253.15.
9. In Konstancin-Jeziorna, Poland, the Sobczynscy family of 5 spends $151.27.
10. In Cuernavaca, Mexico, the Casales family of 5 spends $189.09.
11. In Tingo, Ecuador, the Ayme family of 9 spends $31.55.
12. In Raleigh, North Carolina of the United States, the Revis family of 4 spends $341.98.

Questions directly from the above data:

1. Name the family which spends the least amount on food each week.
2. Name the family which spends the greatest amount on food each week.
3. Name the country which has the largest family size.
4. Name the countries which have the smallest family size.

Sample Questions for further research and discussion.

5. Is the above data a good sample for representing the food consumption around the world?
6. Why is there such a disparity in the food expenditure between different countries?
7. What food items do these countries eat for lunch?

Student activity sheets I, II, III are attached.

Key to to activity sheet II

City	Country	Continent	Latitudes of the cities	Longitudes of the cities
Cairo	Egypt	Asia	30°03ĐN	31°22ĐE
Breidjing Camp	Chad	Africa	45.4983°N	122.691°E
Kodairea	Japan	Asia	35 44'N	139 29' 00'E
Shinkgkhey Village	Bhutan	Asia	27.7°N	91.5° E
Kuwait City	Kuwait	Asia	29°20'N	48°00'E
Bargteheide	Germany	Europe	53°44'N	10°15'E
Sicily	Italy	Europe	37°30'N	14°00'E
Collin Bourn Ducis	UK	Europe		
Konstancin-Jeziorna	Poland	Europe	52°4'60N	4'60N 21°7' 0E
Cuernavaca	Mexico	South America	18°57'N	78°35'60W
Tingo	Ecuador	South America	1°58' ?????	
Raleigh	US	North America	35°52'N	78°47'W

Activity Sheet I

- Complete columns 2 and 3 below based with the information provided above.
- Complete column 4 by calculating the data from columns 2 and 3.
- Complete column 5 by researching on the Internet.

City/Country	Family Size	Price per Week in $	Price per Person per week in $	Average weekly Salary in $
Cairo, Egypt				
Breidjing Camp, Chad				
Kodairea, Japan				
Shinkgkhey Village, Bhutan				
Kuwait City, Kuwait				
Bargteheide, Germany				
Sicily, Italy				
Collin Bourn Ducis, UK				
Konstancin-Jeziorna, Poland				
Cuernavaca, Mexico				
Tingo, Ecuador				
Raleigh, US				

Questions following the table above:

1. What is the least amount a family spends on food per week per person?
2. What is the greatest amount a family spends on food per week per person?
3. Compare the family size, price per week and average weekly salary and draw some conclusions.

- Record the country and continent in which the following cities are located.
- Find the longitude and latitude of the cities and record your answer on the table.

(Hint: Use the Internet or a world map to find the answers.)

City	County	Continent	Latitudes of the cities	Longitudes of the cities
Cairo				
Breidjing Camp				
Kodairea				
Shinkgkhey Village				
Kuwait City				
Bargteheide				
Sicily				
Collin Bourn Ducis				
Konstancin-Jeziorna				
Cuernavaca				
Tingo				
Raleigh				

Activity Sheet III

Mark the continents and location of each of the 12 cities on the map below

Modular Math in Music

RON IMBESSI *Math Teacher, Johansen High School*

Concepts:
Number Sense; fundamental counting principles to compute combinations and permutations; arithmetic series; mathematical reasoning.

Skills:
Counting, writing, ordering, comparing, listening and critical thinking.

Mathematics Standards:
Gr. 8-12: Alg. II 18.0 & 22.0. Grades: 9 – 11

Materials:
- Computer with Microsoft Office, a projector and screen.
- Handout 1 for each student of an alphabetical Mod-26 code to decipher.
- Handout 2 for each student containing two partially filled Mod-12 Grids, one numerical and the other musical.
- Handout 3 for each student containing the songs, "Happy Birthday" and "Jingle Bells", for them to transpose.
- Clear plastic protective sleeves for handouts 2 and 3 (back-to-back in one).
- Dry-Erase pens for each student
- An Instrument (preferably guitar) and a musician to play the transposed songs

Directions:
After a brief introduction and discussion of modular arithmetic, students will pair up with their neighbor to work together on the first handout, using the alphabet (mod 26) to decipher the short phrase, "Math can be way cool." Here, each letter of the alphabet is given a corresponding number in sequence. When a number on the handout is less than 26, the corresponding letter is obvious. But when a number on the handout is greater than 26, the students

The author would like to thank Mrs. Vanisko for her guidance in the writing of this paper.

must start over with the alphabet; 27 becomes "A" again, and so forth to 52 (Z) to complete the second cycle. The goal of this first exercise is for students to discover how simple it is to subtract quantities of 26 from a given number until there remains a positive number less than or equal to 26, from which they can derive a corresponding letter from the key. Upon completion of the 1st handout, opportunity for questions will be given to assure completeness of each student's knowledge of the basics of modular arithmetic.

Then, handouts 2 & 3 will be given in a protective sleeve with the two blank Mod-12 grids on one side and the two songs with chords shown above words at the transition points of the song, on the other. Additionally, dry-erase pens will be handed out at this time to be used on the sleeves for students to show their individual work. The teacher must begin by explaining that unlike the clock, which starts with a '1' and ends with a '12', modular math begins with a '0' as its first value. This fact is not as important when dealing strictly with numbers but will become very important when modulating the musical keys of the chromatic scale. The students will first work together under the teacher's guided instruction to fill in the 1st numerical matrix addition chart and as a class, observe how each successive row and column begins and ends systematically; from 0 to 11, then 1 to 0, then 2 to 1, and so on, revolving back to 0 after 11 each time. Coordinates can then be pointed out, that row 3 intersects with column 5, equaling 8; as row 9 plus column 7 equals 16 (then subtract 12) and it revolves back to 4.

Once the students are generally familiar with the revolving nature in the mod 12 addition matrix, students can then be directed to work separately on the second Mod-12 musical chromatic-scale chart until the majority of students have completed it. Since the 12 tones of music are not as linearly simplistic as twelve sequential steps of the alphabet, in that five of the progressive alphabetical steps will have sharps, the 1st row and column must be provided to guide the modular addition process. For an example of a completed chromatic scale chart, see table 1 below.

After another opportunity to clarify understanding, the students will be asked to turn the handout over and select one of the two songs to practice their new skill on. (See table 2 below for one of the songs.) All students will be assigned to transpose one of the songs in a different key than the one provided and can work individually or pair up with a neighbor. Depending upon the competence of the musician available, the teacher should be sure to at least assign the keys of A, C, D, E, & G to the class, as they are the easy keys for guitarists, and give them 5 – 10 minutes to transpose the chords from their prospective keys to their newly assigned key signature. Finally, the songs will be collected and a musician (whether it be the teacher, a student or a guest from outside the school) will perform the short songs in the transposed keys to test for accuracy. The class will be invited to sing along while the musician plays. Together the class will identify any obvious errors in each successive song and make any adjustments necessary in the new relevant key signature. Depending on the availability of time, the checking for transposed accuracy should be done at least twice per song but no more than four times per song, as relevance may be lost due to too much repetition. This lesson should be easily completed within a standard high school period's allotted time of about 50 minutes.

Tables: Music's 12 Note Chromatic Scales

	+	Key 0	1	2	3	4	5	6	7	8	9	10	11
Key	0	A	A#	B	C	C#	D	D#	E	F	F#	G	G#
	1	A#	B	C	C#	D	D#	E	F	F#	G	G#	A
	2	B	C	C#	D	D#	E	F	F#	G	G#	A	A#
	3	C	C#	D	D#	E	F	F#	G	G#	A	A#	B
	4	C#	D	D#	E	F	F#	G	G#	A	A#	B	C
	5	D	D#	E	F	F#	G	G#	A	A#	B	C	C#
	6	D#	E	F	F#	G	G#	A	A#	B	C	C#	D
	7	E	F	F#	G	G#	A	A#	B	C	C#	D	D#
	8	F	F#	G	G#	A	A#	B	C	C#	D	D#	E
	9	F#	G	G#	A	A#	B	C	C#	D	D#	E	F
	10	G	G#	A	A#	B	C	C#	D	D#	E	F	F#
	11	G#	A	A#	B	C	C#	D	D#	E	F	F#	G

Conclusion:

Assessment of student's participation in all aspects of this lesson is important. Cooperative learning from peer involvement will be essential to those students who don't have a particular interest in music. Though some will certainly choose not to sing with the class, they may be challenged to participate by clapping or tapping on their desks to the beat of the song. Inevitably, it will be humorous when a student makes a transposition mistake and it offers a great opportunity to go back over the modular math to correct the problems. For the young musicians in the class, this lesson will serve as a valuable tool in providing them with a logical and tangible means of altering any song they wish to play, to an alternate key their band finds more suitable. Modular math can take the mystery out of music's key transposition and make both math and music practically applicable, understandable and fun!

References:

Barger, Rita & Haehl, Martha. "Guitars, Violins and Geometric Sequences." Mathematics Teaching in the Middle School (April 2007): 462-466.

Unruh, Kathy. "How to Transpose Guitar Chords." www.abclearnguitar.com/transpose.html.

Happy Birthday

C G
Happy Birthday to you

C
Happy Birthday to you

F
Happy Birthday Dear (So-in-So)

C G C
Happy Birthday to you

Jingle Bells

G C
Dashing through the snow, in a one-horse open sleigh,
D G
O'er the fields we go, laughing all the way,
G C
Bells on bobtails ring, making spirits bright,
D G D
What fun it is to ride and sing a sleighing song tonight, oh

Chorus:

G G7
Jingle bells, jingle bells, jingle all the way,
C G A7 D D7
Oh what fun it is to ride in a one-horse open sleigh, hey,
G G7
Jingle bells, jingle bells, jingle all the way,
C G D G
Oh what fun it is to ride in a one-horse open sleigh

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	X	W	Y	Z
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

39	53	46	86		55	27	66		28	5		50	79	51		29	41	15	38

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	X	W	Y	Z
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

39	53	46	86		55	27	66		28	5		50	79	51		29	41	15	38

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	X	W	Y	Z
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

39	53	46	86		55	27	66		28	5		50	79	51		29	41	15	38

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	X	W	Y	Z
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

39	53	46	86		55	27	66		28	5		50	79	51		29	41	15	38

Matrix Addition - Mod 12

+	0	1	2	3	4	5	6	7	8	9	10	11
0	0	1	2	3	4	5	6	7	8	9	10	11
1	1											
2	2											
3	3											
4	4											
5	5											
6	6						0	1	2	3	4	5
7	7						1	2	3	4	5	6
8	8						2	3	4	5	6	7
9	9						3	4	5	6	7	8
10	10						4	5	6	7	8	9
11	11						5	6	7	8	9	10

0 = The Starting Point 11 = The Ending Point

Music's 12 Note Chromatic Scale

	+	Key 0	1	2	3	4	5	6	7	8	9	10	11
Key	0	A	A#	B	C	C#	D	D#	E	F	F#	G	G#
	1	A#											
	2	B											
	3	C											
	4	C#											
	5	D											
	6	D#						A	A#	B	C	C#	D
	7	E						A#	B	C	C#	D	D#
	8	F						B	C	C#	D	D#	E
	9	F#						C	C#	D	D#	E	F
	10	G						C#	D	D#	E	F	F#
	11	G#						D	D#	E	F	F#	G

0 = A The Starting Point 11 = G# The Ending Point

It's About... Rounding Numbers

MARY GONZALES *Teacher, Caswell Elementary School*

Strand:
Number Sense 1.4

Skills:
Rounding numbers to the nearest ten, hundred, and thousand

Grade:
3

Materials:
Vertical Number Line (see Activity Pages), Numbers for Game.

Description:
In real life, people estimate numbers more often than compute, as exact numbers are not always necessary. One of the basic tools for estimation is the ability to 'round numbers.' Knowledge of place value is a prerequisite for this skill. The multiple steps required to round a number makes learning this difficult as it is first introduced in the third grade. This article describes a successful strategy for third grade students to master the rounding skill using a poem developed by Monica Uskatis of Ceres Unified School District.

Directions:
Introduce the lesson by discussing why real life situations do not always require an exact answer. For example, how long will it take you to clean your room? How many students are in third grade at this school? About how many students altogether go to this school? Elicit more examples from students. Lead the discussion to estimation, using situations that are real to your students and numbers that are near or about the actual scenario.

Objective:
"Today we are going to learn how to round, or estimate numbers". Define rounding in math terms with examples. Explain that rounding facilitates mental math in addition to checking work to determine if the answer makes sense. Reiterate the objective, have students repeat it back, and ask them to write it (in math journals).

The writing of this article was assisted by Dr. Viji Sundar and supported by the Central California Mathematics Project.

Activity 1:

Rounding single-digit numbers. Use the examples from the earlier discussion and start students thinking about rounding and estimating. Next, work Activity Page 1 (see attached) using a number line from 0 to 10. Using a few single-digit numbers as examples, have students decide if the given digit is closer to the 0 or the 10 on the number line. This strategy develops the concept of rounding up or down. Make a chart of the digits closer to 0 as compared to the digits closer to 10. Discuss why five is rounded up. The chart can be used as a reminder of when to round up and when to round down.

Activity 2:

Rounding double-digit numbers. Start by chanting by tens to 100. Explain numbers rounded to tens will be one of these "counting by tens" numbers. A two digit number which is not a multiple of ten will be somewhere between two of the counting by tens numbers. For example, 43 is between 40 and 50. The task when rounding is to determine which of the multiples of 10 is closer to the given number. Students first determine which two tens (or rounded) numbers a given number is between, then decide which is the closer rounded number. Using Activity Page 2 students practice identifying a rounded number using a vertical number line.

In real life, people estimate numbers more often than compute, as exact numbers are not always necessary. One of the basic tools for estimation is the ability to 'round numbers.' Knowledge of place value is a prerequisite for this skill.

General Strategy to Round Numbers

Students need additional skills to round mentally. To teach this skill, introduce the following poem:

Find the rounding place
Look right next door
Four or less just ignore
Five or greater go back
And add one more

STEP 1. *Find the rounding place*
underline the digit in the tens place

STEP 2. *Look right next-door*
draw an arrow from the underline to the ones place

STEP 3. *Four or less just ignore*
retrace your arrow to the tens digit

STEP 4. *Five or greater go back and add one more*
retrace your arrow to the tens digit and increase that digit by one

STEP 5. Change all the digits after the rounding place to zeros because the rounded number only needs to be close to, or an estimation of, the original number

Use Activity Page 1 to visually show the process. If necessary, refer to the chart made earlier to determine if the number rounds up or down. Complete Activity Page 1.

Reinforce *Rounding* with a Game: Create a set of rounded tens numbers from 0 to 100 printed on paper plates or something large enough to be seen by all students. Place them around the room or have students hold them. When given a number, students mentally must round it then race to the correct rounded number. This can be played individually, with partners, or in teams or as a whole class.

Close the lesson with a dialogue on using smart math brains to round or estimate numbers without a number line. Students should write the rounding poem and the steps used to round in a math journal everyday for a week and then again once a week till they gain mastery.

EXTENSIONS:

This technique can be extended to rounding to hundreds and thousands. Expanded notation can be used to show students that when rounding to a place value within a number, the only digits that change are those in the rounding place. By underlining the rounding place, students are able to visualize this concept.

RULES FOR ROUNDING

Find the rounding place
Look right next door
Four or less just ignore
Five or greater go back
And add one more

STEP 1. Find the rounding place—underline the digit in the tens place

43 **58**

STEP 2. Look right next-door—draw an arrow from the underline to the ones place

43 **58**

STEP 3. Four or less just ignore—retrace your arrow to the tens digit

43 **58**

STEP 4. Five or greater go back and add one more—retrace your arrow to the tens digit and increase that digit by one

43 **68**

STEP 5. Change all the digits after the rounding place to zeros because the rounded number only needs to be close to, or an estimation of, the original number

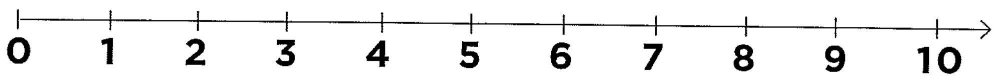
40 **60**

Activity Page 1

Name: ______________________________

Date: ______________________________

Draw a line up and down through the middle number on the number line and answer the questions.



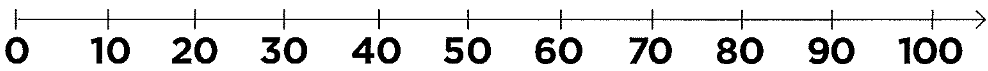
1. Is 4 closer to 0 or 10? __________
2. Is 7 closer to 0 or 10? __________
3. Is 1 closer to 0 or 10? __________
4. Is 6 closer to 0 or 10? __________
5. Is 5 closer to 0 or 10? __________
6. Which digits are closer to 0 than 10? ___________________________
7. Which digits are closer to 10 than 1? ___________________________

Activity Page 2

Name: ______________________________

Date: ______________________________

Plot about where you would find the given number on the number line. Then fill in the numbers in the blanks. The first one is an example for you.



1. Plot the number 9.
   9 is between 0 and 10 but closer to 10.

2. Plot the number 3.
   3 is between _____ and ______ but closer to ______.

3. Plot the number 21.
   21 is between _____ and ______ but closer to ______.

4. Plot the number 43.
   43 is between _____ and ______ but closer to ______.

5. Plot the number 58.
   58 is between _____ and ______ but closer to ______.

6. Plot the number 42.
   42 is between _____ and ______ but closer to ______.

7. Plot the number 86.
   86 is between _____ and ______ but closer to ______.

8. Plot the number 99.
   99 is between _____ and ______ but closer to ______.

Activity Page 3

Name: ______________________________

Date: ______________________________

Round the following numbers to tens:

18 __________

61 __________

85 __________

8 __________

77 __________

12 __________

50 __________

33 __________

98 __________

16 __________

Teaching Basic Math Skills with Excel

STEPHANIE SOUZA, *Math Instructor, Merced College*

BACKGROUND

In 2006 and 2007 a colleague in Humboldt County and I were asked to develop an intensive math course for Forest Service federal employees to prepare them for future data analysis courses. This one-week intensive math "refresher" class included topics from pre-algebra to precalculus. We incorporated Excel projects into the course since Excel was to be their main data analysis tool in subsequent courses. Our audience was mature (ages 40+) with very little to no experience using Excel. This unique group of men and women consisted of:

- people who had not attended school in over 20 years;
- women who had been disheartened by chauvinistic teachers;
- people from a non-technologically oriented generation;
- managers who were used to giving instructions;
- people who had little patience with a subject they had learned over 20 years ago.

The course was based on a series of lessons and project worksheets prepared by the other instructor and me. Though the students brought their own textbook and laptop as references, they obtained most of their information from the lecture, their own discussions, and the small binder filled with Excel worksheets we had prepared.

LESSON STRUCTURE

Each daily session began with a 15 minute lecture, followed by 2-4 problems which reviewed the concepts in the lecture. The students first worked independently, then in groups to discuss the problems and their common errors. Finally, they worked together on problems assigned on the board. The students worked on an Excel worksheet related to the lecture. For these Excel projects, students were allowed to work in a group, but were required to submit their individual solutions.

The writing of this article was assisted by Dr. Viji Sundar and supported by the Central California Math Project.

This article will describe my experience of working with these 20 reentry adults and my experience in incorporating Excel into the lessons. For the two Excel projects presented in this article, I will discuss what Excel offered that "pencil and paper" could not and describe student reactions to each project. Lastly, I will discuss the advantages found from incorporating Excel into the lessons.

EXCEL PROJECT #1 – ORDER OF OPERATIONS

For this project, the students were required to simplify expressions using order of operations. The worksheet consisted of a set of exercises that were arithmetic expressions involving addition, subtraction, multiplication and division (see fig. 1a). The answers were provided and there was space under the problems for students to write the corresponding Excel "code" (i.e. what they would type in Excel to get the answer) as shown in figure 1b.

After typing the "code" into an Excel cell, the students compared their answers to the answers provided on the worksheet. In situations when the answers did not match, the students did some troubleshooting on their own or with a partner to find and correct the error.

Problem	Answers
2^3-6+4	6
$12-30\div5(-3)^2-2$	-44
$\frac{14^3}{6(2-3)}-5^3-\left(\frac{5+4-3}{1-2}\right)$	-576.33333

(a)

Problem	Answers
2^3-6+4 =2^3-6+4	6
$12-30\div5(-3)^2-2$ =12-30/5*(-3)^2-2	-44
$\frac{14^3}{6(2-3)}-5^3-\left(\frac{5+4-3}{1-2}\right)$ =14^3/(6*(2-3))-5^3 -(5+4-3)/(1-2)	-576.33333

(b)

Fig. 1 (a) Worksheet as provided to the students; (b) Completed worksheet with Excel "code".

Observations of Project #1

Writing the Excel "code" on paper was a good start for this group of students. However, the hands on experience of using Excel for order of operations was advantageous as it reinforced the concept of grouping. Students learned that Excel uses the forward slash to signify division, whereas on paper, a horizontal bar is used. I have noticed that in my regular Algebra classes, students forget that the horizontal bar acts as a grouping symbol. Yet, with Excel, the students remembered to group the expression in the numerator and denominator using parentheses. It was necessary for students to think outside the box to consider additional parentheses and proper parenthesis placement in Excel due to the lack of a long horizontal bar symbol in Excel. Excel also clarified the correct use of exponents with negative number bases. A common error students make is not using parentheses when a negative number has an exponent.

To avoid confusion, students were instructed to use parentheses to indicate what is to be done first. For example (–3)^2 versus –(3^2). With Excel, students could play with parentheses placement and compare answers immediately on the screen. Students had to understand order of operations and grouping or else the output would be incorrect.

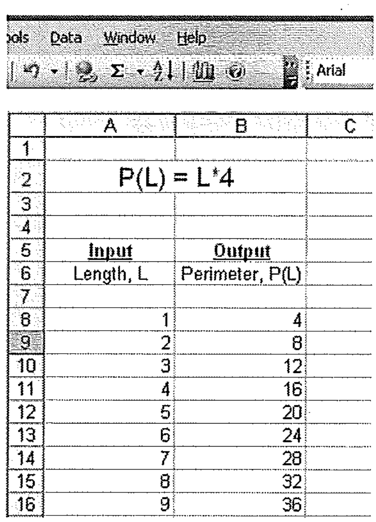
Student Reactions to Order of Operations Project

The students' reaction to this project was mostly positive. They seemed ready to tackle inputting the expressions into Excel because they already had the answers. This helped them focus on the content and to not worry about what the answer should be. The longer expressions, like the third exercise in figure 1a, were difficult for some because there was too much information to easily focus on in one cell. Students more comfortable with Excel, played with breaking up the problem into several cells and then used appropriate operations to get the final answer into a single cell. Most of the students, just like those in traditional classes, became frustrated with the tediousness of finding their errors. Yet, they understood the importance of practice and the need to master this as they may encounter these in projects in subsequent tougher courses.

EXCEL PROJECT #2 - FUNCTIONS

This second project required students to use Excel to graph functions in preparation for data analysis in future courses. We had designed many different graphing problems of functions ranging from linear to exponential. Figure 2 shows an example of a problem in which students were asked to graph the function P(L) = 4L where L represented the length of a side of a square and P(L) is the resulting perimeter. Students began graphing by creating two columns, one labeled input and the other labeled output. The input was length, represented by the variable L. Thus "Length, L" was typed in the input column. The output was perimeter. Thus "Perimeter, P(L)" was typed in the output column.

The entire function, P(L) = 4L, was typed at the top of the page to remind the student of the Excel formula, 4*L, they should type in the output column. In the input column, the students produced a column of sequential integers, usually starting with 1. For every length value in the input column, they copied the formula, 4*L, in the output column to calculate the perimeter of a square. The result is a table of values (see fig. 2) used to create the linear Excel graph displayed in figure 3.



	A	B	C
1			
2	P(L) = L*4		
3			
4			
5	Input	Output	
6	Length, L	Perimeter, P(L)	
7			
8	1	4	
9	2	8	
10	3	12	
11	4	16	
12	5	20	
13	6	24	
14	7	28	
15	8	32	
16	9	36	

Fig. 2 Excel screenshot of perimeter example for functions project.

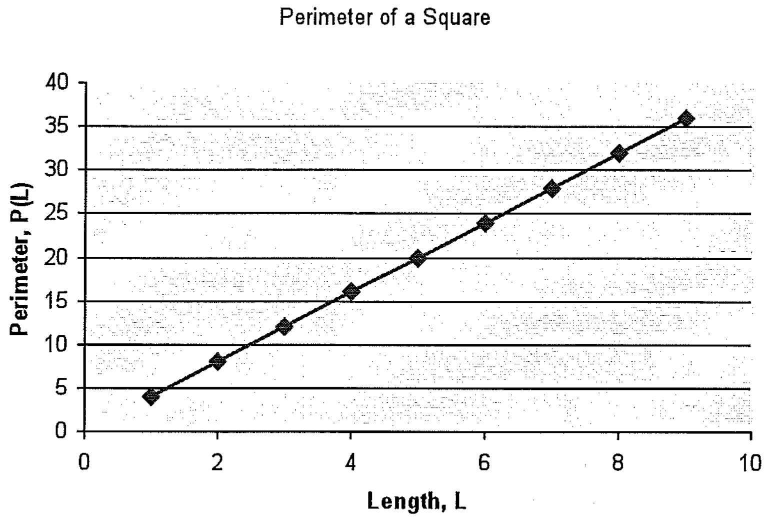


Fig. 3 Excel graph of perimeter of a square function.

Observations of Project #2

There are several ways to explain functions on the chalk board. However, Excel serves as an excellent visual and hands on tool to help students grasp the concept of input and output. These concepts can be conveyed on the board using different colored chalk to represent input and output and providing students with colored pencils. However, this method is not nearly as flexible and quick as Excel. Excel lends itself to trial and error and experimentation. Due to Excel's computational speed and large, colorful graphs, students were able to play with five different functions in the time it would have taken them to sketch one function on paper. In Excel, when the students graphed a parabola they could look at the numeric values in the output column and analyze why different input values resulted in some of the same output values. Furthermore, students could answer questions about the graph's behavior by studying the columns of input and output values as well as the Excel formula they entered.

Student Reactions to Functions Project

At first, students were intimidated by the seeming complexity of functions and input, but with Excel they were able to work without fear and learn how the input values affected the graph. They seemed frustrated when they could not immediately locate their errors. This served as good practice in patience and tenacity. Often students would type the function incorrectly. In time, these became great teaching moments because they broached questions such as, "Why did my graph do that?" One of the students had a large gap in the graph of her parabola, which made it look like a broken bridge. This was a common dilemma among the students both years I taught the course. Thus, I demonstrated how to troubleshoot this problem by analyzing the input and output values in the two columns that corresponded to the approximate location of the gap in the graph. This demonstration enabled the class to work individually or with a partner to find their error. These students discovered that an input value was deleted by accident. A gap or spike in a graph from one incorrect input value and other errors helped students to understand the relationship between inputs and functions.

A by-product of this error analysis was the desire and enthusiasm of many students to understand their graphs. Most of the students would try to determine the reasoning behind a graph's behavior. For example, the students found the graph of an exponential function (see fig. 4) unusual since it seemed to stay horizontal for so long and then spike up. Students studied the graphs as well as the input and output columns and discussed this behavior with their neighbors. The students believed that the flat portion of the graph represented output values of zero. Yet, when they examined the columns of input and output values, they discovered the output was much greater than zero. The students were eager to discuss the reason for the graph's seemingly constant behavior when the output values were not only greater than zero, but increasing as well. Thus, Excel served as a non-threatening auto-tutor on scaling and graph shape.

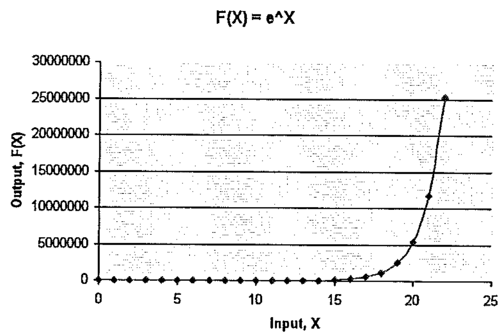


Fig. 4 Exponential function example

ADVANTAGES OF EXCEL

Overall, there were many advantages to using Excel in teaching this basic math skills class. The immediate feedback Excel offered was invaluable to students because they knew right away whether their input was incorrect. Excel errors, such as "#NAME?", are described in the Excel Help so that students can correct the syntax (i.e. Excel "code"), on their own. The feature in Excel that outlines the cell in the same color as the cell name in the formula bar (see fig 5), helped students to identify an input with it's output and become more comfortable with functions. The visual superiority of Excel allowed students to personalize their work with font size preference and color scheme. Also, the ability to display columns of values as well as many graphs on one large screen made analysis easier. The speedy production of graphs made it possible for students to "play" with an idea and extend on it by entering different values of their

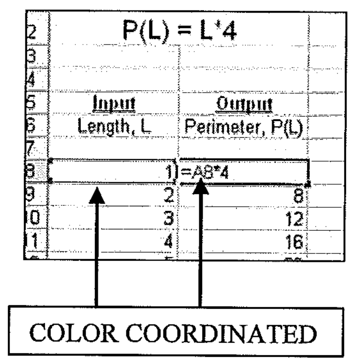


Fig. 5 Color coordination feature in Excel

choice and studying the results. As a tool, Excel served to keep the students engaged in a subject which is typically regarded as dull and boring.

CONCLUSION

Overall, I found that Excel played a significant role in teaching this group of adults to grasp basic math concepts and build confidence. It served as an excellent tool for teaching and reinforcing math concepts while adding fun to an eight-hour long day. In my experience, reentry adults tend to have more frustrations and mental blocks than an average student or child, but they were very receptive to this class. They tried hard and learned several concepts and skills in just one week. They also enjoyed the process and were engaged the entire time. Board work and Excel projects contributed to the class uniquely, but the real benefit to these students was the variety of learning tools and class structure in a long day. Excel brought an eagerness and motivation that was pleasantly surprising. My experience with Excel and these adults leads me to believe that Excel could be beneficial to all ages. If these reentry adults could rise to such a challenge, so could other audiences.

PB&J Sandwich versus Candy Bars: Use Math to Decide

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Concepts:

Number Sense, Measurement and Geometry, Mathematical Reasoning

Skills:

Nutrition label reading, multiplication of decimals, division of decimals, number comparison, monetary manipulation

Mathematical Standards:

Gr 5: NS 1.2, 2.1, 2.2, MR 1.2, 2.3, 2.4
GR 6: NS 1.1, MR 2.4
Gr 7: NS 1.2, MR 2.5

Grades:

5-9

Materials:

In-class activity: Prices and nutritional labels from a jar of peanut butter, a jar of jelly, bag of sliced bread, and an assortment of candy bars; paper; pencil; Student Activity Sheets, pages 1 and 2. Take-home activity: Jar of peanut butter, jar of jelly, package of sliced bread, one candy bar, paper, pencil, Student Activity Sheets, pages 1 and 2.

Background

Nutrition is a daily concern among parents and teachers; indeed even the government tries to influence the nutrition of school children through lunch programs. In this activity, students are asked to compare the nutritional value of a peanut butter and jelly sandwich in relation to a store bought candy bar through calorie and price comparisons. The calories being compared are total calories, calories from fat, calories from carbohydrates, and calories from protein. Depending on the brand names of the items purchased, students may find surprising results in comparing the two foods. In the end, students are asked to choose between the sandwich and the candy bar using their data to justify their choice. This activity further demonstrates the complexities of nutrition and some ways in which mathematics can help in making food choices.

Description

Before starting the activity, students must have a little understanding of calories, fat, carbohydrates, and protein. Calories are a unit of measurement describing the amount of energy a food supplies. The United States Food and Drug Administration bases the percentages on nutritional labels on a 2,000 calorie daily diet, which is about the recommended intake for teenagers. It is interesting to note that, "This level was chosen, in part, because it approximates the caloric requirements for postmenopausal women. This group has the highest risk for excessive intake of calories and fat" (FDA Backgrounder). Fat, carbohydrates, and protein are all compounds found in food which make certain types of caloric contributions. This activity focuses solely on the caloric contribution of these components.

Begin by asking students to write down on a piece of paper the answers to: (1) which would they rather eat: a peanut butter and jelly sandwich or a candy bar (have them choose a specific candy bar), (2) which they believe to be healthier and why, (3) which they believe to have less total calories, (4) which they believe to have less total calories from fat, carbohydrates, or protein, (5) which they believe costs less?

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The first Student Activity Sheet consists of vocabulary words and questions for students to complete before performing the caloric and price comparisons. While both sheets may be completed by the student at home as a project or homework assignment (see the Modifications section for details), I recommend that the first sheet, if used, be a take home assignment. The first section requires students to state the definitions of the relevant nutritional words used. The second section asks 2 questions which set up the comparison assignment. The third section gives 3 bonus questions, the first two of which will require a small amount of research to answer, and the appropriate web-site to find the answers is given. The third question asks students to make a food diary for one day to compute their total caloric intake for that day.

Pass out the Student Activity Sheets. Pass out the nutritional labels and price information for a jar of peanut butter and jelly, a package of bread, and a candy bar to each group or individual student.

Set Up—Food Information:

On the "Food Information: From Nutrition Label" portion of the second Student Activity Sheet, have the students fill in the relevant information from the nutritional label for each food product. For each item they need to record the brand/type of food product, the size of a serving in the package/jar, the number of servings they will use (you may ask them to make a sandwich so they will not have to guess). For instance, when making a whole peanut butter and jelly sandwich, I used 2 tablespoons of peanut butter, 2 tablespoons of jelly, and 2 pieces of bread—for my products, this constituted 2 servings each. Also have the students record the number of calories per serving, grams of fat per serving, grams of carbohydrates per serving, and the grams of protein per serving. At this stage, students may be surprised that their candy bar package may contain more than one serving.

Calorie Comparison:

The "Calorie Comparison" portion of the second Student Activity Sheet is in two parts. On the peanut butter and jelly sandwich chart, have the students compute the total calories per serving, adding up the column to enter the total in the bottom row. Repeat this process for the total calories from fat, carbohydrates, and protein.

On the candy bar chart, have the students compute the total calories, and the total calories from fat, carbohydrates, and protein. Note that the formulas are in the headers of the chart.

Have the students compare the totals. Then ask if they would change their answers of the original first four questions and why.

Price Comparison:

On the "Price Comparison" portion of the second Student Activity Sheet, have the students fill in the price and number of servings used of each food item. Then have the student compute the price per serving and the total price for amount of each food item used. On the peanut butter and jelly sandwich chart, add the entries in the total price for amount of each food item column and write the answer in the bottom box.

Have the students compare the prices, and ask if the results match their guess to the original question (5) on which costs less.

Finally ask students to consider both the calorie comparison and the price comparison and use the information to decide what to eat: a peanut butter and jelly sandwich or a candy bar.

Modifications:

This activity may be modified (without changing the Student Activity Sheets) to be a homework activity requiring students to visit a grocery store or find the information from a grocery store's website. Furthermore, the food items may be changed to any food items with nutrition labels and price tags or any food items for which the calorie, fat, carbohydrate and protein content, and price may be found.

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US FDA/CFSAN, "Make Your Calories Count – Use the Nutrition Facts Label for Healthy Weight Management," <http://www.cfsan.fda.gov/~ear/hwm/labelman.html> (2006).

Student Activity Sheet I

Name:

Definitions

Serving Size:

Calorie:

Fat:

Carbohydrates:

Protein:

% Daily Value:

Attach answers to the following questions.

Questions

1) What is a "Nutrition Facts" label?

2) How can you use the "Nutrition Facts" label to make healthy eating choices?

Bonus Questions

(for the first two questions will need to use information from the Food and Drug Administration at www.fda.gov)

1) The % Daily Value is based on a 2,000 calorie diet. Why?

2) Is a 2,000 calorie diet appropriate for teenagers? If so, why? If not, what is appropriate and how can you use the % Daily Values to make healthy eating choices?

3) Pick one weekday. What is your total caloric intake for this day? (Make a list of each food item and its total calories)

Student Activity Sheet II

Name:

Food Information: From Nutrition Label

	Brand name	Serving size	Total # of servings used	Calories per serving	Fat per serving	Carbs per serving	Protein per serving
Peanut butter							
Jelly							
Bread							
Candy bar							

1) In this activity, you will compare calories, types of calories, and price. Is this enough information to make healthy food choices? Why?

2) If you were to create your own food comparison, what other information from the nutrition labels would you use and why?

Name:

Calorie Comparison

	# servings used	Calories (Cal) # serv. X Cal / serving	Cal from Fat # serv. X fat / serving X 9 Cal / grams fat	% of Cal from Fat Cal from Fat / Cal	Cal from Carbs # serv. X Carbs / serving X 4 Cal / grams Carbs	% of Cal from Carbs Cal from Carbs / Cal	Cal from Protein # serv. X protein / serving X 4 Cal / grams protein	% of Cal from Protein Cal from protein / Cal
Peanut butter								
Jelly								
Bread								
Total								

	# servings used	Calories (Cal)	Cal from Fat	% of Cal from Fat	Cal from Carbs	% of Cal from Carbs	Cal from Protein	% of Cal from Protein
Candy Bar								

Questions:

1) What contributes the most calories from carbohydrates to the sandwich: peanut butter, jelly, or bread? Does this surprise you? Why?

2) Which item has the most total calories, the sandwich or the candy bar?

3) Based upon the above data, which item will you choose to eat? Why?

Student Activity Sheet IV

Name:

Price Comparison

	Price	# servings used	Price Per Serving Total Price / total # servings in package	Price of Amount Used Price per serving X # servings used
Peanut butter				
Jelly				
Bread				
Total				

	Price	# servings used	Price Per Serving	Price of Amount Used
Candy Bar				

Questions:

1) What is the least expensive component of your sandwich? Does this surprise you? Why?

2) Which item costs more the sandwich or the candy bar?

3) Based upon the calorie information and price, which item will you choose to eat? Why?

A Design Model to Help Thai Students Learn Statistics

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The writing of this article was supported by the Central California Mathematics Project.

This article will give an overview of a curriculum model for the course "Statistics for Research" taught to undergraduate students at Phranakhon Rajabhat University (PNRU).

In Phranakhon Rajabhat University (PNRU), Bangkok, Thailand, Statistics is taught to undergraduate students in many non-statistics disciplines, such as chemistry, computer science, biology, and humanity resources management. The statistics courses fulfill the growing demand for students and professionals who should master statistical concepts. Jan and Wisenbaker (2003) reported that teachers at all levels find teaching statistics and probability is immensely challenging. Not only are there new developments in and approaches to the subject matter, but also there are constantly new opportunities afforded by access to innovative instructional materials and methods and more advanced educational technology.

Garfield and Ben-Zvi (2007) stated that the study of statistics provides ideas and methods that could be utilized to better understand the environment at the academic level and for day-to-day understanding of statistical activities. Utts (2003) also emphasizes the importance of statistics education for everyone. Educated citizens should understand basic statistical concepts so that they can detect any misuse of statistics by policy makers, physicians and others.

The Thai government embarked upon substantial educational reform with the 1999 National Education Act (NEA) (Office of the National Education Commission, 1999). The key aspects of this reform focused on improving efficiency and effectiveness of learning. Students were encouraged to become critical and creative thinkers, acquire facility with information technologies, and develop their learning and individual potential based on the philosophy of 'student-centered' learning. In 2002, the Thai government announced plans to install computers complete with Internet connections at all high schools, and make the internet and 'E-learning' or 'Online learning' the technology of choice for the Thai higher education system. (Suanpang and Petocz, 2004)

Why do Students in Thailand Need a New Learning Design?

Research in higher education has shown there is a strong relationship between students' perception of their current learning situation, their previous learning experiences, the manner in which they understand their current learning context, the way they go about studying in that context, and the role of assessment for their learning (Suangpang, Petocz and Reid, 2004). Students find themselves in learning situations, which can both enhance and impair the outcomes of their learning (Prosser and Trigwell, 1999). Petocz and Reid (2003) have shown that students often expect their teachers to provide the enthusiasm for their studies. The impact of this finding is overwhelming "traditional" learning environments which may include face-to-face lectures or tutorials providing learning situations in which students see the value and benefits of being in the classroom. However, "non-traditional" learning environments, including online forums, need to be set up in such away that students see the value and the benefit of participating.

In Thailand, a preliminary study on the business statistics subject using the traditional classroom forum (See above question.) was conducted with undergraduate students of Suan Dusit Rajabhat University, Thailand (Suangpang, Petocz and Reid, 2004). This study showed that, using traditional classroom methods?, the students had a very high failure rate, caused in part by students' difficulty in catching up after missed lectures, and their difficulties in understanding the statistics concepts and applying them to the real world.

PNRU offers a 'Statistics for Research' course for undergraduate students. The objective of the course is to provide basic statistics knowledge required for conducting research. The traditional teaching was one three-hour lecture per week, in which students learn statistical theory but without any practice session. However, in the second aspect of the course, a computer-based practice statistics package was offered as a separate class. In the last facet of the course, students conducted research projects in their fields. In this research project component, some students found it difficult to put statistical principles into practice. This highlighted the need for a new teaching strategy to improve the students' learning outcomes.

The Proposed Design to Help Students Learn Statistics

There have been several studies about teaching statistics and students' learning. For example, Siddiqui, Yeo and Zadnik (2002) indicated there is no single unique teaching and learning strategy that can produce complete student satisfaction in the classroom. Franklin and Garfield (2006) reported that, as a rule, teachers of statistics should rely much less on lecturing, and much more on alternatives such as projects, laboratory exercises, group problem solving, and discussion activities. Groen and Carmody (2005) indicated that blended learning seeks to introduce students to the diverse environment and experiences of professional practice. Driscoll (2002) asserts that "blended learning means different things to different people, which illustrates its widely untapped potential". The Mathematics Department of PNRU decided upon defining the concept of blended learning as the integrated combination of pedagogical approaches to produce an optimal learning outcome, coordinated with instructional technology focusing on skill-driven and intended learning.

This approach to blended learning provides different learning styles that integrate in-class and out-of-class learning. The blended learning design has the following components:

1. Lecture with Discussion: 'Statistics for Research' meets once a week for two hours. These lectures serve to convey, clarify, explain, extend, and emphasize the concepts that form the content knowledge. In order to motivate students' interest and encourage critical thinking, forums for discussion are provided.
2. Computer Laboratory sessions: A two-hour computer laboratory session using SPSS for Windows is provided once a week. The computer laboratory aims at developing students' knowledge and skills in application of statistical methods.
3. Fact-to-Face Workshop sessions: Fact-to-face workshop sessions are arranged once a month. The goals of these sessions are: (I) to provide opportunities for students to present their research project progress, (II) to develop collaborative student teams, and (III) to encourage students to become critical and creative thinkers in asking and giving suggestions to other groups. The time provided for each workshop depends on the number of research groups. In general, there are 10 minutes for presentation and 10 minutes for questions and suggestions for each research group.
4. Online and Face-to-Face Discussion with the lecturer: Opportunities are provided for online (via email) and face-to-face discussion between students and the lecturer to discuss not only their group work but also their self-directed learning and paper-based exercises.
5. Research Project Group Work: For the duration of the semester, students are engaged in an out-of-class collaborative group activity which is a research project. Each research project group consists of 4-5 students. Students are allowed not only to choose their own teammates but their preferred research topic as well. The research projects are conducted over the period of 16 weeks during which the students are enrolled in the Statistics for Research course. In the last week of the semester, students present their research results in a conference. The research project group work focuses on developing statistical knowledge and its application and also on developing skills in effective communication and collaboration.

Assessment of Students' Learning

The Mathematics Department of PNRU discussed what students should know and be able to do after completing the Statistics for Research course. The department concluded that it is not enough for students to understand content knowledge, they should also know the relevant skills as well. According to the learning design, the assessment of the knowledge and skills objectives of the subject should be blended. Thus, the following assessment scale was developed:

- Assessment of computer laboratory exercises (worth 15%)
- Assessment of research project group activity with emphasis on the development of teamwork skills, written and oral presentation skills, and the students' practical skills in the implementation of statistical methodology (worth 15%)
- Assessment with two midterm examinations. Time provided for each midterm exam is 100 minutes (worth 20% each)
- Assessment with a two-hour final exam (worth 30%)

Each examination is closed books and closed-notes.

Observations on the New Learning Design

After implementing the new design, we observed that the students spent more hours studying statistics out-of-class than in a traditional classroom. It is necessary to ask whether the time, both practicing statistics in a computer laboratory and doing research project group work outside the regular lecture, was well spent. The percentages, means, and standard deviation for the questionnaire on students' views on computer laboratory (Prasitkusol, 2006) are presented in Table 1.

The students (n=83) showed positive response to the computer laboratory practice for all items excluding item 5, "I feel nervous when I have to learn new procedures on a computer". Although they were nervous about learning new procedures on the computer, they enjoyed learning with computers (item 8) and disagreed with the statement on avoiding using the computer (item 6). In addition, the students believed the laboratory enabled them to have a better understanding of statistical concepts (item 4). All things considered, the students were "very satisfied" with the computer laboratory practice (item 15).

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Table 1 Questionnaire items with percentages, means and standard deviations according to Likert scale (1 = strongly disagree, 2 = disagree, 3 = neutral, 4 = agree, 5 = strongly agree)

	Item/ Statement	**Students' Views**					
		Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree	Mean (SD)
1	All the tasks I completed in the computer laboratory helps me to learn.	33	**61**	5	1	0	4.26 (0.61)
2	Having the demonstrations helped me to understand how statistics can be put into practice	21	**68**	9	1	1	4.05 (0.68)
3	The lecturer encouraged me to actively participate in the computer laboratory.	**46**	44	10	0	0	4.41 (0.84)
4	The computer laboratories helped me understand statistics concepts better.	30	56	11	3	0	4.13 (0.71)
5	I feel nervous when I have to learn new procedures on a computer	5	29	40	19	7	3.31 (1.50)
6	If I can avoid using a computer, I will.	1	6	9	44	40	1.86 (0.92)
7	Computer laboratory helped to develop my problem solving skills.	30	47	19	4	0	4.09 (0.98)
8	Using a computer makes learning more enjoyable.	35	47	18	0	0	4.17 (0.71)
9	I have a lot of self-confidence in using computer.	14	53	29	4	0	3.78 (0.74)
10	I understand the relationship between computer laboratories and lectures.	13	60	19	8	0	3.85 (0.97)
11	I feel more confident that I have understood concepts.	8	46	36	10	0	3.51 (0.79)
12	There was enough time in the computer laboratories to complete all the tasks.	4	42	25	23	6	3.21 (1.22)
13	The lecturer was available to discuss my difficulties I encountered.	33	50	12	4	1	4.10 (0.85)
14	The way I learn in statistics helps me study in other courses.	12	40	43	4	1	3.64 (1.01)
15	All things considered, I was satisfied with the Computer laboratories.	18	60	18	4	0	3.92 (0.72)

In addition to the multiple-choice questionnaire, students responded to the open-ended questions: (i) what was helpful for learning (ii) what might be improved. *(This statement is redundant.)* Responses to question (i) included: computer laboratory practicing (9), "SPSS helps me understand statistics concept better" (1), "I learn better because statistics package help me to analyze a large set of data easily" (1), research project (5), and "the teaching and content of the subject help me learn how to conduct the research" (1). Responses to question (ii) included: "more time practicing in the laboratory helpful", and "more examples might increase understanding."

Conclusion

Statistics courses present an opportunity to integrate professional skills into the college curriculum. Reid and Petocz (2004) stated that tertiary students in a wide range of areas need such skills for their future professional lives. Studies of students' understanding of professional work and their conceptions of learning are summarized in the notion of Professional Entity.

It is neither easy to design nor to implement a new course design to enhance students' learning. However, the results of the experimental design at PNRU have been encouraging. Here are a few reasons why learning designs should be continually modified or changed:

- To motivate students' interest
- To meet the needs of different learning styles
- To provide learning environments where students see the value and benefit of participating
- To offer extra support to students who need special assistance

Although altering the learning design can support and assist students to learn more effectively, some lecturers will not change their teaching method because of:

- Inadequate computer skills
- Unavailability of a computer laboratory
- Difficulty in procuring mathematics and statistics packages because of license fees
- The extra time and effort needed to prepare new materials using a new design.

We are aware that it is challenging to design a new model to enhance students' learning of mathematics and statistics that will be embraced by all faculty. However, the PNRU mathematics department feels that this new teaching and learning strategy will improve student learning. This learning design has sparked excitement in the department to keep the new format and continue to improve it.

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Quartiles Made Easy By An Excellent Formula

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The writing of this article was supported by the Central California Mathematics Project.

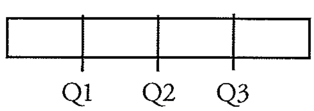
In Thailand, the mathematics curriculum for high school students consists of content in algebra, geometry, trigonometry, logic, matrix algebra, real numbers, complex numbers, and probability and statistics.

In particular, descriptive statistics is offered to high school students. Descriptive statistics frequency distributions and graphs, measures of central tendency, measures of dispersion and measures of position, all of which contain many formulas students need to know. Instructors try to simplify some formulas to make student understanding easier. Measures of position such as quartiles, deciles and percentiles all use the similar formulas. Ordinarily, students need to know the position of quartiles, deciles, or percentiles before finding their specific values. In this article, we will explore a new process to find these values.

Consider the problem:

A father asked his four kids how many cuts must he make in a chocolate bar so that it is evenly shared between the four of them, Three cuts or four cuts?

A picture will help us determine the answer.



Oh! Three cuts for four kids

First idea in term quartiles: Quartiles are numbers that partition, or divide an ordered data set into four equal parts.

WHY DO STUDENTS NEED TO LEARN QUARTILE

Quartile are a step in learning measures of position. Sutdents are frequently confused with the formula below that is used in Thai textbooks. Quartile r-th:

$$L+(\text{ class width})\{\frac{rN}{4}-\text{cumulative frequency of the Qr interval}\}/\text{Frequency of the Qr class}$$

Where L = the lower boundary of the Qr interval

And N = the total number of items. An alternative way of writing down this formula is

$$Q_r = L + I\left[\frac{\frac{N}{2} - \sum f_l}{fQ}\right]$$ Where: Q_r is the r-th quartile, $\frac{rN}{4}$ = the index of quartiles

$\sum f_l$ = cumulative frequency before Q_r class. fQ = frequency of Q_r class.

The easy way to calculate the quartiles is the Difference Ratio method that I will show.

MEASURE OF POSITION

Quartiles can be calculated by several methods, such as using a formula, calculated by Microsoft Excel or SPSS that are popular in America.

In Thailand, Thai students like a manual calculation or trick to save time. Most students need to take an entrance exam for the university after finishing their high school. The national test is provided for all Thai University. Only 25% of all students can study in the closed universities per year. Also every student needs to complete the entrance test in short time, so they need the quick calculation for each topic.

This article presents a trick to calculate which can be easily understood.

A quartile's position, or index, is labeled as: the first quartile index (IQ1), the second quartile index (IQ2) and the third quartile index (IQ3). Just as in the chocolate bar example, there are three positions or cuts for four parts.

DEFINITION 1:

The three quartiles Q_1, Q_2 and Q_3 approximately divide an ordered data set into four equal parts. About one quarter of the data falls on or below the first quartile Q_1. About one half the data falls on or below the second quartile Q_2, and about three quarters of the data falls on or below the third quartile Q_3. The second quartile is the same as the median of the data set (Larson and Farber, 2000).

Let N be the number of items in the ordered data set. Since, quartile 1, Q_1, is the data item which is larger than 25% of the data, or one quarter of the way through, IQ_1 is $\frac{N+1}{4}$. Likewise, quartile 2, Q_2, the median, is the 50% mark of the data, so IQ_2 is the number $\frac{N+1}{2}$. Finally, quartile 3, Q_3, is the 75% mark of the data, or three quarters of the way through the data. So IQ_3 is number $\frac{3(N+1)}{4}$. These index numbers IQ_r are used to determine the quartiles, Q_r.

Remark: $IQ_2 = 2(IQ_1)$ and $IQ_3 = 3(IQ_1)$

Students could easily be confused here between the measure itself and the position of the measure. Some rewording of the above might help.

Here is an example to illustrate finding the first, second and third quartiles of a data set?

EXAMPLE 1

Data set 'A' :23 :18 :20 :29 :10 :8 :15 :13 :22 :18

Given the list below, find the value of the data "items" of the quartiles.

The table 1:

Order Data 'A'	8	10	13	15	18	18	20	22	23	29
Data position	1	2	3	4	5	6	7	8	9	10

N=10

The table 2:

Index IQ_r	$IQ_1 = \frac{10+1}{4} = 2.75$	$IQ_2 = \frac{n+1}{2} = 2(IQ_1) = 5.5$	$IQ_3 = \frac{3(n+1)}{4} = 3(IQ_1) = 8.25$
Item	Q_1	Q_2	Q_3

Solution: Index IQ_1, is 2.75 is in the interval (2, 3) so Q_1 is between the corresponding data 10 and 13

$2 < 2.75 < 3$: $\{10 \leq Q_1 \leq 13\}$ To find the first quartile, use the difference ratio:

$$\frac{Q_1 - 10}{13 - 10} = \frac{2.75 - 2}{3 - 2}$$ Solve $Q_1 = 3(0.75) + 10$. Thus, $Q_1 = 12.25$

In the same way, we also can solve Q_2 and Q_3.

CONSTRUCTING A FREQUENCY DISTRIBUTION FROM A DATA SET

Creation of new Trick:

Suppose the index $\mathbf{IQ_r}$ is between the integers n and n+1, so the quartile $\mathbf{Q_r}$ is between the data a and b

$\{n \leq \mathbf{IQ_r} \leq n+1\}$: $\{a \leq \mathbf{Q_r} \leq b\}$ The difference ratio relates all the information:

$$\frac{IQr - n}{n + 1 - n} = \frac{Qr - a}{b - a}$$

Solve the difference ratio for Q_r, to obtain: $Qr = (b - a)(IQr - n) + a$

FINDING THE QUARTILES IN THE GROUPED DATA CASE:

How do we use Quartiles to specify the position of a data entry within a data set?

The data set is partitioned into intercals of equal width. The following forumlas apply to the resulting frequency distribution:

1. Find the median class i.e. the interval in which the $\frac{N}{2}$ middle item Q_2 lies.The position of Q_2 is called IQ_2

2. N is the number of data in a data set. It is also the sum of the of frequencies for a data set, $N = \sum f$. Quartile 1, Q_1 is the 25% mark of the data, one quarter of the way through the data, so Q_1 is about item$\frac{N}{4}$ in the list. The position of Q_1 is called IQ_1.

   Quartile 3, Q_3, is the 75% mark of the data, three quarters of the way through the data, so Q_3 is about item $\frac{3(N)}{4}$ in the list, so third position of Q_2 is called IQ_3. (Lane, D. 2008).

   Remark: $IQ_2 = 2(IQ_1)$ and $IQ_3 = 3(IQ_1)$

EXAMPLE 2

Data set 'B', the following sample data set, lists the number of minutes 50 internet subscribers spent on the internet during their most recent session. Construct a frequency distribution that has seven classes; find the value of the data "items" of the quartiles.

50 40 41 17 11 7 22 44 28 21 19 23 37 51 54 42 88 41 78 56 72 56 17 7 69 30 80 56 29 33 46 31 39 20 18 29 34 59 73 77 36 39 30 62 54 67 39 31 53 44 (N=50)

Solution: The Table 4

Grouped data set 'B'

Step 1: Create the boundary classes and cumulative frequency.

Class	Boundary Class	Frequency, f	Cumulative frequency, cf.
7–18	6.5–18.5	6	6
19–30	18.5–30.5	10	16
31–42	30.5–42.5	13	29
43–54	42.5–54.5	8	37
55–66	54.5–66.5	5	42
67–78	66.5–78.5	6	48
79–90	78.5–90.5	2	50
		$\sum f = 50$	

Step2: Find the position of quartiles and check class intervals

$$IQ_1 = \frac{N}{4} = \frac{50}{4} = 12.5$$

12.5 is between cumulative frequency 6 and 16, so Q1 is in the boundary class 18.5 to 30.5

$\{18.5 \le Q_1 \le 30.5\} : \{6 \prec 16\}$, Now use the difference ratio to find Q1:

$$\frac{Q_1 - 18.5}{30.5 - 18.5} = \frac{12.5 - 6}{16 - 6}$$

Thus, $Q_1 = 26.3$

Now solve for Q2: IQ2 = 2IQ1 So that IQ2 = 2(12.5) = 25.

In the same way we can find Q2,and Q3

The trick is to build a frequency distribution of the data in the generalization:

Now the Excellent Formula may be created. First some notational definitions:

Qr is the r-th quartile;

Lb is a lower bound of a frequency class; Ub is an upper bound of a frequency class

IQr is the index of the r-th quartile;

Cfl is the cumulative frequency immediately less than IQr ; Cfr is the cumulative frequency immediately greater than IQr

Next, use the difference ratio:

Let: $D_1 = IQ_r - cf_l$, $D_2 = cf_r - cf_l$ and $I = Ub - Lb$

Now rewrite the difference formula and solve for Q:

$\frac{Q - Lb}{Ub - Lb} = \frac{IQ_r - cf_l}{cf_r - cf_l}$ Thus, $Q_r = \frac{D_1}{D_2} I + Lb$ is called the

Excellent- Formula

HOW TO INTERPRET FRACTILES SUCH AS PERCENTILES?

Percentiles and Other Fractiles:

In addition to using Quartiles to specify a measure of position, you can also use Deciles and Percentiles. These common fractiles are summarized as follows.

The Table 5

Fractiles	Summary	Symbols
Medians	Partition a data set into two equal parts.	Q2
Quartiles	Partition a data set into four equal parts.	Q1, Q2, Q3
Deciles	Partition a data set into ten equal parts.	D1, D2, D3, …,D9
Percentiles	Partition a data set into one hundred equal parts.	P1, P2, P3, …, P99

Remark: All these articles also can use the trick formula too.

Reference:

Larson&Farber. (2000). Elementary Statistics, Prentice-Hall, Inc. Upper Saddle River, New Jersey 07458

Lane,D. (2008). Measures of Central Tendency, Under a Creative Commons Attribution License on April 20, http://cnx.org/content/m11061//latest/.

Don't Wear That Button Out!

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In recent years, technological tools like scientific calculators have left an indelible mark on the teaching of mathematics. Many instructors have integrated technology into their courses as a means of reinforcing concepts and illustrating the power of these classroom aids.

When students use a scientific calculator in the classroom, they will inevitably begin to play with it. For example, they may press a particular button over and over again. On older calculators this corresponds to applying the same operation repeatedly. With more modern calculators, repeated operations must be performed manually. The purpose of this paper is twofold: we will examine what happens when certain calculator buttons are repeatedly pressed, and then we will explain the results mathematically.

Square Root Iterations

Nearly every calculator, from a grocery calculator to a programmable graphing calculator, contains a square root button. Let us pick several initial numbers at random and repeatedly press the square root button. Clearly, we select initial values which are positive. The results are shown in Table 1. Each table's left column, n, indicates how many times the square root button has been pressed thus far, and the right column, x_n, lists the screen output. Eight digits are given since many standard calculators can only display that many digits. The randomly chosen initial numbers are listed in the row where $n = 0$.

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n	x_n
0	1945
1	44.102154
2	6.6409453
3	2.5770032
4	1.6053047
5	1.2670062
6	1.1256137
7	1.0609494
8	1.0300240
9	1.0149010
10	1.0074229
⋮	⋮
20	1.0000072
⋮	⋮

n	x_n
0	0.3792
1	0.6157922
2	0.7847243
3	0.8858466
4	0.9411943
5	0.9701517
6	0.9849628
7	0.9924529
8	0.9962193
9	0.9981079
10	0.9990535
⋮	⋮
20	0.9999991
⋮	⋮

n	x_n
0	42
1	6.4807407
2	2.5457299
3	1.5955344
4	1.2631446
5	1.1238971
6	1.0601401
7	1.0296311
8	1.0147074
9	1.0073268
10	1.0036567
⋮	⋮
20	1.0000036
⋮	⋮

Table 1: Three square root sequences: $x_n = \sqrt{x_{n-1}}$.

Notice that in each table the sequence of numbers in the right column appears to approach 1. Now let us prove that, given any positive initial value, the square root sequence does in fact converge to 1.

Observe first that all of our sequences can be generated by the recursive formula

$$x_n = \sqrt{x_{n-1}} \quad \text{for } n \geq 1.$$

If our initial value $x_0 = 1$, then the sequence is constant and obviously converges to 1. Hence, there are two cases to consider.

Case 1

Let us assume that $x_0 > 1$. Since the square root function is increasing on its domain, we have

$$x_1 = \sqrt{x_0} > \sqrt{1} = 1.$$

Iterating this process,

$$\begin{aligned} x_2 &= \sqrt{x_1} &= \sqrt{\sqrt{x_0}} &> \sqrt{1} &= 1, \\ x_3 &= \sqrt{x_2} &= \sqrt{\sqrt{\sqrt{x_0}}} &> \sqrt{1} &= 1, \end{aligned}$$

and so on. By an inductive argument (details left to the reader), we can conclude $x_n > 1$ for all $n \geq 0$. Moreover, each term in the sequence can be expressed in terms of the initial value as

$$x_n = \sqrt[2^n]{x_0}. \qquad (1)$$

Thus, we know

$$\sqrt[2^n]{x_0} > 1. \qquad (2)$$

To establish convergence of the sequence, we need to prove the following statement:

$$1 \le \sqrt[2^n]{x_0} \le \sqrt[n]{x_0} \le \sqrt[n]{n} \quad \text{for } n \ge x_0.$$

The first inequality is (2), while the last inequality follows from the fact that $\sqrt[n]{x}$ is a strictly increasing function of x and $n \ge x_0$. To justify the center inequality, observe that $2^n > n$ for all $n \ge x_0$. Because $x_0 > 1$, we have $\sqrt[2^n]{x_0} \le \sqrt[n]{x_0}$, and the entire statement follows.

If we recall that $\lim_{n\to\infty} \sqrt[n]{n} = 1$ from analysis [4], then we may apply the squeeze theorem to conclude $\lim_{n\to\infty} \sqrt[2^n]{x_0} = 1$. Thus,

$$\lim_{n\to\infty} x_n = \lim_{n\to\infty} \sqrt[2^n]{x_0} = 1,$$

and therefore the square root sequence converges to 1 for an initial value $x_0 > 1$.

Case 2:

Now assume $0 < x_0 < 1$. Clearly, $\dfrac{1}{x_0} > 1$. Applying the result from Case 1 to the initial value $1/x_0$, it follows that

$$\lim_{n\to\infty} \sqrt[2^n]{\frac{1}{x_0}} = 1. \qquad (3)$$

Then we have

$$\lim_{n\to\infty} x_n \overset{(1)}{=} \lim_{n\to\infty} \sqrt[2^n]{x_0}$$

$$= \lim_{n\to\infty} \sqrt[2^n]{\frac{1}{1/x_0}}$$

$$= \lim_{n\to\infty} \frac{1}{\sqrt[2^n]{\frac{1}{x_0}}}$$

$$\overset{(3)}{=} 1.$$

Therefore, the square root sequence also converges to 1 if the initial value $0 < x_0 < 1$, and the proof is complete.

Logarithmic Iterations

Let us examine what happens when we begin with a positive number and repeatedly press the ln (natural logarithm) button on a scientific calculator. Several starting numbers were selected at random, and the natural logarithm button was repeatedly pressed. The results are shown in Table 2.

n	x_n
0	56.02
1	4.025709
2	1.392701
3	0.331245
4	−1.104897
5	error

n	x_n
0	106.28
1	4.666077
2	1.540319
3	0.4319893
4	−0.8393544
5	error

n	x_n
0	7425018
1	15.82037
2	2.761298
3	1.015701
4	0.01557890
5	−4.161838
6	error

Table 2: Three logarithmic sequences: $x_n = \ln(x_{n-1})$.

Observe that a negative number was attained in each sequence, causing the process to terminate. We encourage the reader to try other starting values and determine what happens. Are there initial values for which the logarithmic sequence converges?

To answer this question, first express the sequence mathematically as

$$x_n = \ln(x_{n-1}), \text{where } n = 1,2,3,\dots \qquad (4)$$

Suppose this sequence of points were to converge to some number, say X. Taking the limit of both sides of (4) as $n \to \infty$, we have

$$\lim_{n\to\infty} x_n = \lim_{n\to\infty} \ln(x_{n-1}).$$

Since $\ln(x)$ is a continuous function on its domain, we move the limit inside the logarithm to get

$$\lim_{n\to\infty} x_n = \ln\left(\lim_{n\to\infty} x_{n-1}\right).$$

Substituting $X = \lim_{n\to\infty} x_n = \lim_{n\to\infty} x_{n-1}$, we now have

$$X = \ln(X). \qquad (5)$$

Clearly, we must have $X > 0$ for $\ln(X)$ to exist. Equation (5) is equivalent to

$$e^X = X, \qquad \text{or} \qquad e^X - X = 0. \qquad (X > 0)$$

Define $f(x) = e^x - x$. Note that $f(0) = 1$. Differentiating $f(x)$ yields

$$f'(x) = e^x - 1,$$

which is nonnegative for all $x \geq 0$. Thus, f is nondecreasing for $x \geq 0$, meaning

$$f(x) \geq f(0) = 1 \text{ for all } x \geq 0.$$

Therefore $f(x) \neq 0$ for $x \geq 0$, and so $e^x \neq x$ for $x \geq 0$. Equivalently, $x \neq \ln(x)$ for all $x > 0$, and we can conclude the logarithmic sequence in (4) does not converge regardless of its initial value.

Many calculators also include a log button for the common logarithm. We encourage the reader to check that similar results can be obtained in the base 10 case, and more generally, when a logarithm of any other base is used instead.

Trigonometric Iterations

In this section, we will examine what happens if a student repeatedly presses the same trigonometric button. Most trigonometry students first work with angles measured in degrees, so we will start with that.

A random initial number was selected, and the sine button was repeatedly pushed, generating an infinite sequence of numbers using the recursive formula $x_n = \sin(x_{n-1})$. Although the output of the sine function has no units, when we apply the sine function again, we are implicitly assigning the "degrees" units to the new input. Three initial selections and their corresponding results are shown in Table 3, with eight digits displayed as before.

n	x_n
0	15.23
1	0.2626944
2	0.004584867
3	8.002102×10^{-5}
4	1.396630×10^{-6}
5	2.437580×10^{-8}
6	4.254379×10^{-10}
⋮	⋮

n	x_n
0	−396.037
1	−0.5883076
2	−0.01026772
3	-1.792056×10^{-4}
4	-3.127727×10^{-6}
5	-5.458914×10^{-8}
6	$-9.527603 \times 10^{-10}$
⋮	⋮

n	x_n
0	557
1	−0.2923717
2	−0.005102827
3	-8.906113×10^{-5}
4	-1.554410×10^{-6}
5	-2.712957×10^{-8}
6	$-4.735003 \times 10^{-10}$
⋮	⋮

Table 3: Three sine sequences, in degrees: $x_n = \sin(x_{n-1})$.

Repeating the process for the cosine button, we generate data tables using the recursive formula $x_n = \cos(x_{n-1})$. Again, random initial values were used, and we implicitly assign units of "degrees" to our new inputs. Table 4 shows three examples.

n	x_n
0	1165
1	0.08715574
2	0.9999988
3	0.9998477
4	0.9998477
5	0.9998477
⋮	⋮

n	x_n
0	67.1
1	0.3891240
2	0.9999769
3	0.9998477
4	0.9998477
5	0.9998477
⋮	⋮

n	x_n
0	−105.64
1	−0.2695922
2	0.9999889
3	0.9998477
4	0.9998477
5	0.9998477
⋮	⋮

Table 4: Three cosine sequences, in degrees: $x_n = \cos(x_{n-1})$.

In each of these experiments, it appears all the sine sequences quickly converge to zero, and all the cosine sequences rapidly converge to a number close but not equal to one, as indicated in Tables 3 and 4. Similar results occur when working in radians. The sine sequences appear to again converge to zero, albeit more slowly, as shown in Table 5. The cosine sequences seem to converge to a different number in an oscillatory fashion, which can be seen in Table 6 and Figure 1. A line has been drawn in Figure 1 at the value to which the cosine sequences appear to converge.

n	x_n
0	328
1	0.9563847
2	0.8171128
3	0.7291731
4	0.6662532
5	0.6180448
6	0.5794428
7	0.5475577
8	0.5206036
9	0.4974039
10	0.4771456
⋮	⋮
25	0.3235761
100	0.1694877
250	0.1084713
500	0.07704608
1000	0.05461422
⋮	⋮

n	x_n
0	−2.78
1	−0.3537643
2	−0.3464315
3	−0.3395435
4	−0.3330567
5	−0.3269332
6	−0.3211402
7	−0.3156487
8	−0.3104332
9	−0.3054711
10	−0.3007425
⋮	⋮
25	−0.2489704
100	−0.1555647
250	−0.1045526
500	−0.07560497
1000	−0.0540940
⋮	⋮

n	x_n
0	1.41
1	0.9871001
2	0.8344313
3	0.7409147
4	0.6749631
5	0.6248685
6	0.5849906
7	0.5521916
8	0.5245543
9	0.5008273
10	0.4801514
⋮	⋮
25	0.3244840
100	0.1696157
250	0.1085047
500	0.07705804
1000	0.05461848
⋮	⋮

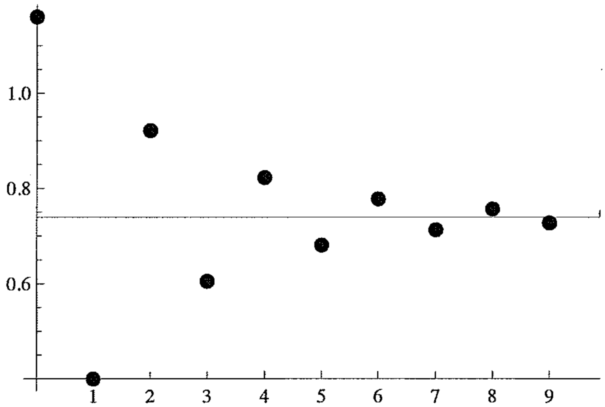
Table 5: Three sine sequences, in radians: $x_n = \sin(x_{n-1})$.

n	x_n
0	1.16
1	0.3993395
2	0.9213180
3	0.6047710
4	0.8226323
5	0.6802942
6	0.7773877
7	0.7127483
8	0.7565676
9	0.7271964
10	0.7470411
⋮	⋮
25	0.7390639
50	0.7390851
100	0.7390851
⋮	⋮

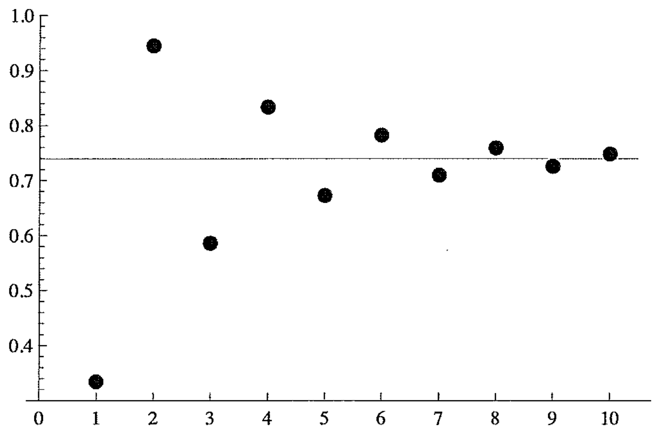
n	x_n
0	847
1	0.3342222
2	0.9446657
3	0.5860138
4	0.8331519
5	0.6725466
6	0.7822377
7	0.7093380
8	0.7587932
9	0.7256669
10	0.7480570
⋮	⋮
25	0.7392582
50	0.7390851
100	0.7390851
⋮	⋮

n	x_n
0	−16.5
1	−0.7023971
2	0.7632958
3	0.7225616
4	0.7501142
5	0.7316110
6	0.7440991
7	0.7356984
8	0.7413622
9	0.7375493
10	0.7401188
⋮	⋮
25	0.7390824
50	0.7390851
100	0.7390851
⋮	⋮

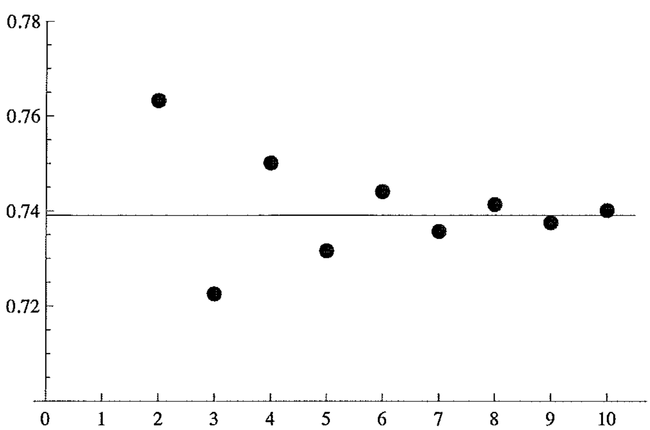
Table 6: Three cosine sequences, in radians: $x_n = \cos(x_{n-1})$.



$[x_0 = 1.16]$



$[x_0 = 847]$



$[x_0 = -16.5]$

Figure 1: Data plots for the three cosine sequences from Table 6.

Do the sine and cosine sequences always converge, regardless of their initial values or units of measurement? If so, to what values do they converge? If not, are these results merely a trick played on us by the calculator? We encourage the reader to try their own initial values and see if their results corroborate with ours.

Let us start with the sine sequences, which are more straightforward to discuss. Recall that these sequences are generated recursively by the formula

$$x_n = \sin(x_{n-1}), \text{ where } n = 1, 2, 3, \ldots \qquad (6)$$

If the initial value x_0 is an integer multiple of π, then the remainder of the sequence consists entirely of zeroes. This means we only must consider values of $x_0 \neq k\pi$, where $k \in \mathbb{Z}$.

Recall that for any angle θ, whether in radians or degrees, we have the inequality

$$-1 \leq \sin(\theta) \leq 1.$$

So regardless of the angle measurement and the initial value, if $x_0 \neq k\pi$ for $k \in \mathbf{Z}$, then x_1 must lie inside $[-1,0) \cup (0,1]$. Moreover, if $x_1 \in (0,1]$, then every number in the sequence after x_1 will also fall in $(0,1]$; if $x_1 \in [-1,0)$, then x_2, x_3, and so on, are found in $[-1,0)$. To summarize, for all $n > 1$, either every x_n is positive or every x_n is negative.

Suppose that $x_n > 0$ for all $n > 1$. Recalling the fact that $\sin(x) < x$ for all $x > 0$ [1], we can write

$$x_{n-1} > \sin(x_{n-1}) = x_n.$$

Thus, we have a strictly decreasing sequence of numbers in the interval . Consequently, the sequence must converge, regardless of units.

On the other hand, if we have $x_n <$ for all $n > 1$, then we can apply the fact that $\sin(x)$ for all $x < 0$ [1] to get

$$x_{n-1} < \sin(x_{n-1}) = x_n.$$

This time, we have a strictly increasing sequence of numbers found in $[-1,0)$, which again means the sequence converges, regardless of units.

But to what number does the sine sequence converge? Let us say the limit of such a sequence is Y. From our previous work,

$Y \in [-1,1]$. Because the sine function is continuous on its domain, we conclude that Y satisfies the equation

$$Y = \sin(Y).$$

Hence, the limit of the sine sequence is a solution of the equation $x - \sin(x) = 0$, regardless of units. An obvious solution is $x = 0$, so let us see if others exist. Define $g(x) = x - \sin(x)$ on the domain $[-1,1]$. We restrict our attention to $[-1,1]$ because Y must be in the range of the sine function. Differentiating g produces

$$g'(x) = 1 - \cos(x),$$

which is positive on $[-1,0) \cup (0,1]$, and is zero when $x = 0$. Thus, g is increasing on $[-1,0) \cup (0,1]$. Since g is continuous everywhere, it follows that g is increasing on all of $[-1,1]$. Hence, g can have at most one root in the interval $[-1,1]$. But we have already found a root at $x = 0$, so we conclude that the sequence given in (6) must converge to $Y = 0$ in degrees or radians.

The question of convergence for the cosine sequences shall be answered with a different approach. As with the sine sequences, we express the cosine sequences with the recursive formula

$$x_n = \cos(x_{n-1}), \text{ where } n = 1, 2, 3, \ldots \tag{7}$$

Since $\cos(x) \in [-1,1]$ for all x, we are assured that $x_1 = \cos(x_0)$ will be in $[-1,1]$ for any x_0, regardless of the units. Moreover, if $x_{n-1} \in [-1,1]$, then $x_n = \cos(x_{n-1})$ lies in $[0,1]$. Therefore, $0 \le x_n \le 1$ for all $n \ge 2$. From this fact, we find that

$$0 \le \frac{x_n + x_{n-1}}{2} \le \frac{1+1}{2} = 1, \text{ for } n > 2.$$

Because $\sin(x)$ is an increasing function on $[0,1]$ in both degrees and radians, we conclude

$$0 \le \sin\left(\frac{x_n + x_{n-1}}{2}\right) \le \sin(1), \text{ for } n > 2. \tag{8}$$

Next, recall the difference-to-product identity

$$\cos(A) - \cos(B) = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right). \tag{9}$$

Using (7) and (9) together, for $n > 2$, we have

$$| x_{n+1} - x_n | = | \cos(x_n) - \cos(x_{n-1}) |$$

$$= \left| -2\sin\left(\frac{x_n + x_{n-1}}{2}\right)\sin\left(\frac{x_n - x_{n-1}}{2}\right)\right|$$

$$= 2\left|\sin\left(\frac{x_n + x_{n-1}}{2}\right)\right|\left|\sin\left(\frac{x_n - x_{n-1}}{2}\right)\right|.$$

Applying the bound in (8), we obtain

$$| x_{n+1} - x_n | \leq 2\sin(1)\left|\sin\left(\frac{x_n - x_{n-1}}{2}\right)\right|. \qquad (10)$$

Since $|\sin(x)| \leq |x|$ for all angles x, equation (10) becomes

$$| x_{n+1} - x_n | \leq 2\sin(1)\left|\frac{x_n - x_{n-1}}{2}\right| = \sin(1)\, | x_n - x_{n-1} |.$$

Because $0 < \sin(1) < 1$, both cosine sequences are contractive and hence convergent [1]. Let us call the limit Z, whose value does depend on whether we work in radians or degrees. Since the cosine function is continuous on its domain, Z is a solution of the equation $x = \cos(x)$. An exact solution to this transcendental equation cannot be written down. However, our earlier data tables provide us the first eight digits of Z in radians and degrees.

Conclusion

One central idea links together all of the sequences we have presented. Each situation has resulted in an equation of the form $x = f(x)$, where $f(x)$ is the function representing the operation of the calculator button which was repeatedly pressed. Solutions of such equations are mathematical invariants called *fixed points*. In terms of sequences, fixed points are the initial values that produce constant sequences. Searching for fixed points occurs in many areas of mathematics, typically in any instance where the solution of an equation must be found but cannot be done by standard algebraic methods. For example, fixed points of Newton's method correspond to roots of a given function [2]. Moreover, fixed points and iterated functions serve as a springboard into the topic of fractals, which are accessible to students of almost any background [3, 5]. In closing, we also encourage the reader to look for fixed points of other functions by repeatedly pressing other calculator buttons.

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Linear and Abstract Algebra for Teachers
A non-Calculus-based Course

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Background for the Course

In December 2002, the California Commission on Teacher Credentialing (CCTC) expanded the guidelines to teach mathematics in secondary schools to ameliorate the shortage of mathematics teachers qualified to teach secondary and middle school mathematics. According to a 2002 NCES (National Center for Education Statistics) report, 37% of high school math teachers and 69% of middle school level of math teachers lacked a major or certification in their field. This statistics of shortage of qualified mathematics teachers is alarming when the expectations for what students should know in these subjects are rising.

In order to mitigate this problem of math teacher shortage, CCTC created a bifurcation credential for subject matter competence—the Full Credential and the Foundation Level Credential. Consequently, it provided two main paths to become a secondary or middle school mathematics teacher. The mathematics content knowledge to enter the credential program could be met either by coursework or by passing a series of Examinations called CSET—California Subject Examination (in mathematics) for Teachers. (www.cset.nesinc.com/CS_testguide Mathopener.asp). The level of competence to teach is to be based on the coursework completed (if coursework path is chosen) or the number of examinations passed (if examination path is chosen). There are three CSET examinations for teachers—CSET 1 (Domain 1 & 2 Algebra & Geometry), CSET 2 (Domain 3 & 4—Number Theory and Probability and Statistics), and CSET 3 (Domain 5—Calculus) http://www.cset.nesinc.com/CS_SMR_opener.asp

Rationale for the Course

The topics for the subject matter competence –also known as Subject Matter Requirement (SMR)—are clearly spelled out by CCTC (www.ctc.ca.gov/educator-standards/). The Subject Matter Requirement for Algebra includes concepts involving matrix algebra, groups, rings, and fields – topics that are usually covered in mathematics courses which require at least two semesters of calculus. As such, for those who are not majoring in mathematics, finding a math course on topics involving linear and especially abstract algebra is extremely difficult, if not impossible.

How the Course Was Developed

It is not surprising to see that the SMRs for Foundation Level Credential include topics in linear and abstract algebra. Beyond simple arithmetic, linear algebra is one of the most widely used mathematical subjects today. The linear algebra concepts intertwine with the topics of high school algebra at its core, and form a bridge between high school algebra and abstract algebra. The school math textbooks, even at the elementary levels, reinforce the axioms (properties) of the whole numbers and the real number system, which are best illustrated in the basic concepts of abstract algebra.

Our original plan was to cover the needed material in two separate courses—one in linear algebra and one in abstract algebra—parallel to a regular undergraduate curriculum. This natural sequencing allows that concepts in linear algebra be used as examples in abstract algebra. This plan had to be abandoned, as there was no guarantee that teachers would take these classes in succession. So we decided to develop a one semester four-unit-credit course that will subsume the topics of SMRs with the concepts of both courses.

Beyond simple arithmetic, linear algebra is one of the most widely used mathematical subjects today. The linear algebra concepts intertwine with the topics of high school algebra at its core, and form a bridge between high school algebra and abstract algebra.

Involvement of all Stakeholders

We consulted with our colleagues in the mathematics department to assure the mathematical integrity of the course and to begin the process of making the course part of the regular departmental offerings. Next we engaged in a dialogue with teachers to get their input as the course being designed was specifically for teachers. In summer 2004, we invited future teachers and current classroom teachers for two Saturday sessions to get feedback on their perspectives on such a course, and to explore the readability of materials we considered using for the course. We wanted to ensure that our course was geared at the appropriate level. The teachers were given a stipend for their participation. These sessions were very beneficial in fine tuning the course topics and the structure of the course offerings.

The Course Core

Linear systems of equations are used to investigate Gaussian elimination, matrix inverses, and determinants. These linear systems provide rich examples of the role played by matrices that do not possess inverses and have zero determinants. Additionally, they lend themselves to an introduction to sensitivity analysis using the inverse of the coefficient matrix. With Markov processes, even eigenvalues and eigenvectors are introduced.

Abstract algebra begins with symmetry groups, and, in particular, the symmetries of a square. This visual perspective relates transformational geometry with matrix multiplication and sets the stage for the concept of a group. Applications in coding theory using modular arithmetic further extend the concept of a group and lead naturally into rings and fields. Permutation groups are studied briefly. The development of the real number system from counting numbers through real numbers is seen in the context of groups, rings, integral domains, and fields.

As the course content was discussed, it became apparent that this course would not only meet the needs of future middle/junior and high school teachers, it could also serve as a mathematical content course for current elementary teachers wanting to teach mathematics at the 6–8 level. As this course is less theoretical and focuses more on contemporary applications of the topics covered, it is listed in the catalog as a lower division mathematics class with pre-calculus as the prerequisite. After we researched and discussed the topics to be included in the course, we arrived at the Course Syllabus—see attachment

We consulted with our colleagues in the mathematics department to assure the mathematical integrity of the course...we engaged in a dialogue with teachers to get their input as the course being designed was specifically for teachers.

Sample Lesson

Linear algebra began with vectors, both from algebraic and geometric perspectives. In geometric projections, the role of the dot or scalar product in linear regression and correlation are used as examples. Students were amazed to learn that the correlation coefficient for two sets of data is the cosine of the angle between the vectors. Throughout the course, examples were grounded in real life applications and tapped into students' previous knowledge forming a bridge to the new concepts introduced.

The following example demonstrates the manner in which we give students a rationale for vector operations. We selected an example associated with ordering food from a fast food restaurant. Each entry under a given name is a one-dimensional array representing the quantity of each specific item to be ordered, so these must be kept separate from one another, but the one-dimensional arrays taken together can form a matrix. Using this, students learn to distinguish between rows and columns. Taking the product of the appropriate "place an order" matrix with the column vector representing the costs of each item, respectively, results in a vector representing the costs for each individual. This allows students to see multiplication of a vector by a matrix as successive applications of the dot product.

Example: Fast Food Orders

Motivate

- Multiplication by a scalar
  Double a person's order; each entry in that column is doubled.
- Add vectors
  Add together the orders of the three people to determine how many of each item must be prepared.
- Dot or scalar product
  For each person, multiply the number of each item and add up the results to determine the cost of each person's order.

items	Kim	Tom	Sal	cost $
hamburger	8	5	2	3.60
fries	6	5	1	1.50
soda	4	5	0	.90
milk shake	2	0	1	2.40

- Multiplication of a vector by a matrix
  Switch the array of person's orders to rows instead of columns and take the product of the matrix with the column vector representing the costs. Each entry and the overall dimensions make sense.

$$\begin{bmatrix} 8 & 6 & 4 & 2 \\ 5 & 5 & 5 & 0 \\ 2 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \$3.60 \\ \$1.50 \\ \$.90 \\ \$2.40 \end{bmatrix} = \begin{bmatrix} \$46.20 \\ \$30.00 \\ \$11.10 \end{bmatrix}$$

Course Materials and Technology

Finding appropriate materials for teaching this course was difficult, especially for the abstract algebra portion. We selected a very special book, *Principles and Practice of Mathematics* (ISBN: 0-387-94612-8), that was developed through the Consortium for Mathematics and its Applications (COMAP). In particular, we chose two chapters from this text, Chapter 3 on Linear Algebra, authored by Alan Tucker, and Chapter 9 on Abstract Algebra, authored by Joseph Gallian. These two chapters provide exceptionally clear explanations for a person with a pre-calculus background, and they contain valuable examples and exercises. The abstract algebra portion was supplemented with materials on rings and fields.

In our pilot offering of the course, even students who were unfamiliar with Excel at the start were working well with the program in a short period of time.

In the linear algebra portion of the course, the use of technological tools is virtually essential. Although *Mathematica* or Maple would be ideal, such software is not readily available to middle school and high school teachers. Graphing calculators work well, but we opted for spreadsheet technology, since nearly every teacher and classroom has access to a spreadsheet program such as Excel. Many of the commands used in linear algebra are functions named in Excel; there are just a few variations in the manner in which some of these are executed. In our pilot offering of the course, even students who were unfamiliar with Excel at the start were working well with the program in a short period of time.

Assessment

Assessment had three components: take-home exams, shorter in-class exams, and a final project. The culminating project required the students to develop a *teaching module* involving topics covered in the course that would be appropriate for the grade level they are currently teaching or expect to teach. The lesson had to comply with both the NCTM standards and the California mathematics content standards for the grade level targeted. Students had the option of working alone or in pairs, and the final report had both an oral component and a written component. In the pilot offering, participating teachers chose a wide variety of applications, including having young children determine the day of the week they were born using modular arithmetic and having high school students investigate the connection between scores on the California High School Exit Exam and school course grades.

Conclusion

It was not easy to get mathematics department approval for math content courses for classroom teachers. The first offering of *Linear and Abstract Algebra for Teachers* was as a special session class through University Extended Education. However, this course received full approval as a catalog offering from the University-wide Education Policy Committee. It is a lower division course (Math 2670) and is intended for both pre-service and in-service teachers who are not mathematics majors but who want to have the proper authorization to teach mathematics at either the middle school or high school level in California. In the pilot course offering the teachers enrolled in the course found the material challenging, interesting, and yet not overwhelming. At the start, few felt that the topics covered would relate to what they taught, but, by the end, each person found an application that was associated with his or her grade level.

Course Outline

(Meeting for 4 hours per week for 14 weeks)

Part I Vectors (Week 1)

- Review of Sine and Cosine Functions
- Vectors and Coordinate Systems
- Vector Algebra and the Dot Product

Part II Linear Algebra (Weeks 2 - 8)

- Linear Systems as Models
- Basic Operations on Vectors and Matrices
- Matrix Multiplication
- Gaussian Elimination
- Matrix Inverses
- Determinants and Vector Cross Product
- Introduction to Eigenvectors and Eigenvalues
- Angles, Orthogonality and Projections

Part III Abstract Algebra (Weeks 9 - 12)

- Symmetry Groups
- Abstract Groups
- Coding Theory
- Introduction to Permutation Groups
- Rings and Integral Domains
  (Integers and Integers Modulon and Matrices)
- Fields of Rational, Real, and Complex Numbers and Polynomials Over a Field

Part IV Course Wrap-up and Final Presentations
(Weeks 13-14)

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