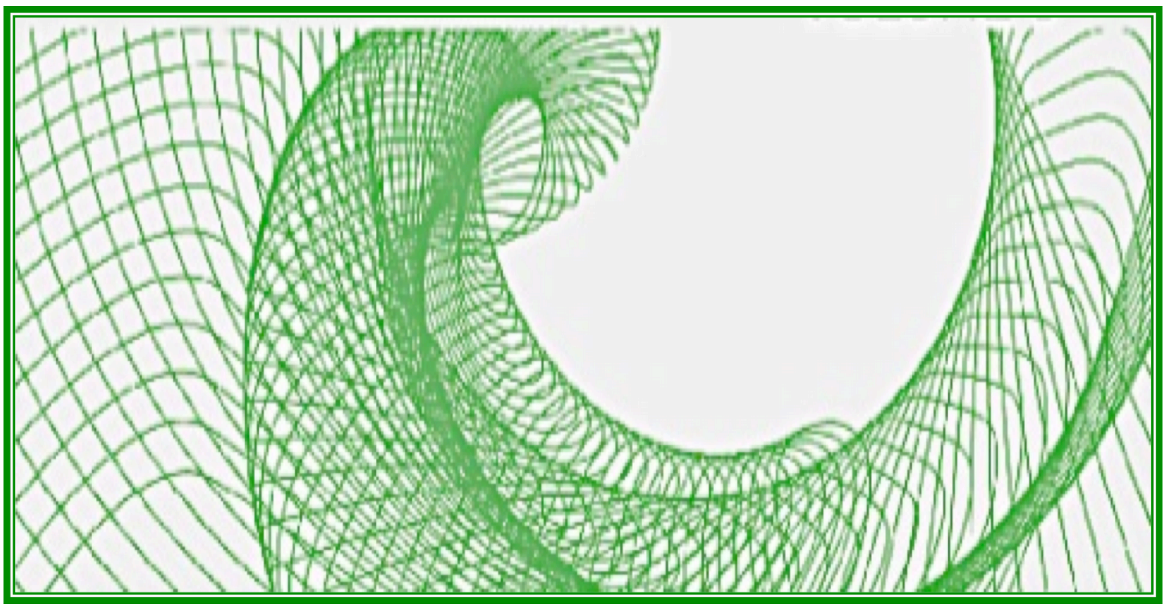
JOURNAL OF THE CALIFORNIA MATHEMATICS PROJECT



CALIFORNIA STATE UNIVERSITY STANISLAUS

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Dedicated to

Dr. Susie Häkansson

Executive Director of the

California Mathematics Project

INTRODUCTION

In this issue of the California Mathematics Project Journal (CMPJ) we continue to ask teachers for their favorite ways of organizing and presenting mathematics lessons at any level: elementary, middle, high school and beyond. We also look for exemplary professional development programs as well as pertinent research papers.

The reader will find a wide range of interests in the articles in this journal. In this issue, one article describes a program designed to instill more confidence in K-12 students when solving STEM problems based on the art of animation. Students are taught the basic skills of cartoon animation; they are taught to break down complex motions of characters down to a sequence of simpler motions.

This issue also contains research-based article appropriate to classroom practices and professional development programs. One of the articles discusses the role a student's autobiography plays in the mathematics teaching-learning process. Another article describes the history and impact of the California Mathematics Project in the professional development of California mathematics teachers. In particular, the article focuses on the San Joaquin Valley Mathematics Project.

Finally, this issue contains a history of mathematics article that delights the calculating impulse mathematics folks get more often than they would admit; this is a great idea for introducing the laws of exponents and logarithms.

This issue is dedicated to Susie Häkansson, the Executive Director of the CMP.

Viji K. Sundar, Editor
Department of Mathematics
California State University, Stanislaus
vsundar@csustan.edu

Angelo Segalla, Editor
Department of Mathematics
California State University, Long Beach
asegalla@csulb.edu

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TEACHER LEADERSHIP DEVELOPMENT AS A CRITICAL COMPONENT OF SYSTEMIC REFORM: THE SAN JOAQUIN VALLEY MATHEMATICS PROJECT

CAROL FRY BOHLIN. CALIFORNIA STATE UNIVERSITY, FRESNO

SUMMARY: The California Mathematics Project (CMP) is a statewide network of university-based sites that are united by a common mission of mathematics leadership development for K-12 teachers. This chapter traces the history of the CMP, placing its expansion in the 1980s and early 1990s within the context of a progressive education movement in California that significantly impacted all facets of education—curriculum, instruction, assessment, and professional development. A detailed description of one CMP site, the San Joaquin Valley Mathematics Project, provides a glimpse into how that site structures a variety of experiences and professional opportunities to help strengthen teachers' competence in mathematics content and pedagogy, their knowledge of issues related to mathematics education, and their capacity and skills as educational leaders. The chapter concludes with a discussion of the recent "back to basics" movement in California and its impact on the California Mathematics Project.

Since its inception in 1982, the California Mathematics Project (CMP) has sought to develop the mathematics skills and leadership capabilities of K-12 teachers through intensive summer institutes and a variety of academic year programs sponsored by regional CMP sites. This chapter focuses on ways in which one of the fifteen regional CMP sites, the San Joaquin Valley Mathematics Project (SJVMP), facilitates and supports the development of mathematically competent, professionally knowledgeable, and actively involved teacher leaders. To provide a meaningful context for discussing the SJVMP, the chapter first provides a brief historical overview of the educational and political environment within which the CMP was established and in which it developed and flourished during its first 15 years.

CALIFORNIA'S DECADE OF SYSTEMIC, PROGRESSIVE SCHOOL REFORM: In 1983, *A Nation at Risk* was published, stimulating a national effort to improve K-12 education. By the mid-1980s, California's Department of Education–rallying under the enthusiastic leadership of State Superintendent of Public Instruction Bill Honig–had initiated the establishment of a coordinated, systemic reform of curriculum, instruction, assessment, and professional development. For the next decade, California was widely viewed as a leader in progressive education. The major elements of California's systemic reforms are described below, with an emphasis on the professional development component and with particular focus on mathematics.

CURRICULUM AND INSTRUCTION: Between 1985 and 1992, curriculum frameworks were developed and published by the California Department of Education for each of the subject matter areas required for high school graduation. Each of these guides was influenced by research, professional judgment, and documents produced by professional organizations such as the National Council of Teachers of Mathematics. The frameworks promoted "critical thinking and conceptual understanding; problem solving based on real-life problems; meaning-centered rather than memorization-oriented learning; active learning and activity-based instruction; contextualized learning which makes connections to students' experiences; collaborative learning in groups, and interdisciplinary learning" (Intersegmental Coordinating Council, 1993, p. 3).

In 1991, the California Department of Education sponsored the development of *Seeing Fractions*, the first of a number of mathematics "replacement units" designed to (a) provide an example for textbook publishers of curriculum aligned with the 1992 *Mathematics Framework* and to (b) provide teachers with the first of many *Framework*-aligned units to replace or supplement chapters in current mathematics textbooks. The state also sponsored numerous workshops throughout California to familiarize 5th-grade teachers with *Seeing Fractions* and with the new instructional expectations contained in the new *Mathematics Framework*. Curriculum writing flourished during the early 1990s. Numerous replacement units and integrated high school mathematics courses were developed—all focusing on promoting students' mathematical reasoning and understanding, and all incorporating hands-on activities, technology, problem solving, writing, performance assessment, and collaborative learning. A number of teacher leaders were involved in the writing, field testing, and implementation of these curricular units/courses. Materials approved during the 1994 California K-8 mathematics textbook adoption featured many innovative curricula that were developed in accordance with the Instructional Materials Criteria included in the 1992 *Mathematics Framework*.

To complement the new subject matter frameworks, four seminal reports covering the K-12 grade span were produced by task forces convened by the California Department of Education (CDE): *Here They Come, Ready or Not!* (1988); *It's Elementary!* (1992); *Caught in the Middle: Educational Reform for Young Adolescents in California Public Schools* (1987) and *Second to None: A Vision of the New California High School* (1992). These documents promoted a "rich, meaning-centered, thinking curriculum," emphasizing reform initiatives such as developing "students' higher order thinking skills…[as opposed to utilizing] the conventional linear lockstep approaches" (Intersegmental Coordinating Council, 1993, p. 21). Teacher networks (e.g., the California Alliance for Elementary Education and the California High School Network) supported the dissemination, application, and implementation of the reform ideals presented in these documents.

ASSESSMENT: Another component of the state's systemic reform efforts was the development of a new assessment system aligned with the progressive tenets of the curriculum frameworks. In 1991, Senate Bill 662 (Hart) established the California Learning Assessment System (CLAS), replacing the California Assessment Program (CAP). State Superintendent of Public Instruction Bill Honig believed that the new performance-based test would create opportunities for students to demonstrate their understanding of the subjects tested rather than assessing how well they could guess on a traditional multiple choice test. From 1991 to 1993, CLAS developed and field-tested the mathematics, science, history–social science, and English–language arts tests for grades 4, 5, 8, and 10. The mathematics portion incorporated open-response items and a relatively small number of multiple choice items designed primarily to assess a student's conceptual understanding, number sense, and ability to solve application problems, as opposed to his or her computational proficiency. In 1993 and 1994, thousands of teachers received instruction on how to use and design rubrics to assess the open-response items on the CLAS tests (and similar assessments). In 1994, over 3000 teachers scored more than three million pieces of student work. Many teachers considered the CLAS instruction to be an extremely important and enlightening professional development experience. The focus on analyzing students' solutions to non-routine problems provided teachers with insight into their students' reasoning and their understanding–or lack of understanding–of mathematics concepts. (The assessment system was not without controversy, however, especially from proponents of a more traditional accountability system. In September, 1994 Governor Pete Wilson vetoed a bill that would have authorized the continuation of CLAS, thus eliminating the program.)

PROFESSIONAL DEVELOPMENT–THE CALIFORNIA SUBJECT MATTER PROJECTS: The final element of California's systemic reform plan in the 1980s and early 1990s was the establishment of a statewide professional development system to provide a coordinated approach to staff development and to build teacher capacity to implement the pedagogy and content of the curriculum frameworks (Carlos & Kirst, 1997). This professional development system became known as the California Subject Matter Projects (CSMPs).

CSMP HISTORY (1981-1998): The original model for the CSMPs was the Bay Area Writing Project (BAWP), which held its first summer institute in 1974 at the University of California, Berkeley. This institute drew together expert teachers from all grade levels to examine critically the research on writing, to practice and hone their own writing skills, and to share their most effective techniques for teaching writing (University of California Office of the President, 1994). In 1981, funding was provided by the California legislature to establish a statewide network of sites based upon the BAWP model, creating the California Writing Project (CWP). The following year, the University of California supported legislation (SB 424) that instituted the California Mathematics Project (CMP). By the end of 1983, eight regional CMP sites had been established (California Postsecondary Education Commission, 1986).

In 1987, a state-commissioned evaluation of professional development concluded that in general there was little coherence or systematic planning in the inservice education that teachers received in California, and that staff development was fragmentary and tended to support traditional school structures and teaching strategies (Little, Gerritz, Stern, Guthrie, Kirst, & Marsh, 1987). The report named two notable exceptions—the CWP and the CMP, which were recognized as containing program elements that led to significant changes in teachers' thinking and practice.

In response to this study, legislation was proposed and passed in 1988 (SB 1882–the Professional Development Act) that established five more California Subject Matter Projects for subjects required for high school graduation. A fiscal augmentation in 1992 led to the expansion of existing sites, the development of new sites, and the creation of two new CSMPs for a total of nine projects (Arts, Foreign Language, History—Social Science, International Studies, Literature, Mathematics, Physical Education—Health, Science, and Writing). The goal (though rarely met) was to establish sites throughout the state so that no California teacher would have to drive more than 100 miles to attend CSMP-sponsored activities. A recent report (Inverness Research Associates, 1998) noted that the CSMPs "provide direct support to over 67,000 teachers every year" and that "one-fifth of all California teachers are involved in at least one Subject Matter Project each year."

CSMP GOALS: The CSMPs have been administered by the University of California under the direction of a Concurrence Committee since 1988. In 1990, this committee developed a set of guiding principles for all of the projects. In summary, projects were expected to (a) use a "teachers teaching teachers" model, where instructional and content expertise was shared among participating teachers; (b) hold 2-4 week summer institutes and a series of academic year follow-ups focusing on subject matter content and the latest ideas in the discipline and their classroom applications; (c) foster and sustain teacher leadership by supporting curriculum development and article writing, encouraging leadership in regional and statewide professional associations, providing guidance in grant development and research, and providing and promoting opportunities for leadership at the project site and on school, district, county, and/or state curriculum and instruction committees; and (d) maintain a regional, discipline-based network consisting of K-12 teachers and college/university faculty (Bartels, 1990; Intersegmental Coordinating Council, 1993).

A major aspect of the CSMPs, therefore, is to develop and sustain subject matter area teacher leaders. Bohlin (1999) has characterized *teacher leaders* as those teachers who have made a commitment to improving their knowledge of [a particular subject matter area] and exemplary instructional practices and actively engage in helping other teachers to do the same. Little (1988) has stated that "it is increasingly implausible that we could improve the performance of schools…without promoting leadership in teaching by teachers" (p. 78). Accordingly, a recent survey of the CSMPs stated that these "teacher leaders are the horsepower for educational reform," serving in seven primary roles: (a) workshop presenters in schools and districts, (b) members of school site leadership teams, (c) leaders for local school restructuring efforts, (d) district or state-designated mentor teachers, (e) team teachers or peer coaches in their schools, (f) subject matter specialists for local schools, and (g) curriculum developers for schools and districts (Stokes, Hirabayashi, & St. John, 1998).

The California Mathematics Project: The CMP network is comprised of fifteen regional sites based at either a University of California or a California State University campus. Faculty members serve as the sites' Principal Investigators, with a majority also serving as Project Directors. Each regional site is expected to provide leadership in mathematics education to the K-12 schools located in the site's service area. This goal is primarily accomplished through the development of a cadre of teacher leaders with the knowledge, confidence, and skills to provide leadership at their school sites and in their districts.

Although each site is uniquely administered and defines its own goals and objectives, all sites subscribe to a set of common beliefs–e.g., the importance of mathematics competency; the belief that all teachers are capable of becoming more knowledgeable, confident, and competent mathematics educators; the belief that teachers have the capacity for leadership and must play a central role in leading efforts to improve education; and the vision of mathematical power for all students (California Mathematics Project, 1994). Sites "create a professional home for teachers that is based upon a culture of inquiry, experimentation, and reflection" (Medina & St. John, 1997, p. iv) that extends throughout the life of the project. Each CMP site hosts a two- to four-week initial summer institute for an average of 30 new participants, and most sites also sponsor a "Tier II" leadership institute for project "alumni." A variety of academic year follow-up experiences (e.g., study groups, action groups, workshops, conferences, and electronic networks) nurture the development of teacher leadership among site participants. The CMP model is one of long-term, sustained professional development, in contrast to a conference or a "one-shot" workshop. Experienced and interested teacher leaders often assume leadership roles at the Project site, becoming site co-directors, institute coordinators, study group leaders, regional coordinators, or institute instructors. They are also given many opportunities to assume local, regional, and/or statewide leadership roles in mathematics education via involvement in professional organizations (e.g., the California Mathematics Council and its affiliates) and in activities related to the state's curriculum, instruction, and assessment initiatives.

This paper has provided a brief summary of California's progressive reforms of the 1980s and early 1990s with a focus on the California Subject Matter Projects. The next section provides a more in-depth look at one site of the California Mathematics Project–the San Joaquin Valley Mathematics Project (SJVMP)–to examine how that site develops and sustains teacher leaders. Quotations from Summer Institute journals and from post-Institute assessments are woven into the description to help paint a picture of the SJVMP experience from the perspective of participating teachers.

The San Joaquin Valley Mathematics Project: The San Joaquin Valley Mathematics Project is based at California State University, Fresno (CSUF), and serves teachers from five counties throughout California's Central Valley—a vast 22,405 square mile agricultural area consisting of 162 school districts, over 700 schools, and over 350,000 students who collectively speak over 100 different languages. Initial funding for the SJVMP was received in 1988. The staff consists of (a) a Project Director/Principal Investigator and two Co-Principal Investigators who are CSUF faculty members, (b) a Coordinator of Professional Development, (c) three Regional Coordinators, and (d) twelve Grade Level Leaders. All of the non-university staff are classroom teachers or curriculum specialists who have assumed leadership positions in the Project after being involved with the Project for at least a year. (More details may be found in the section on Academic Year Support.) The sections that follow provide details about the Summer Institutes and the academic year support for participants, with a focus on the ways the SJVMP helps to develop teacher leadership.

INITIAL SUMMER INSTITUTE: Each year since 1989, approximately thirty-five K-12 teachers and administrators have applied for and been accepted to participate in the two-week SJVMP Summer Institute and in a variety of follow-up experiences. Due in part to the geographic expansiveness of the service area, the first week of the Summer Institute is residential, a model that has proven to be especially powerful in facilitating professional growth and for developing close, supportive collegial relationships. The second week of the Summer Institute is nonresidential. Participants commute to a site near the geographical center of the SJVMP service area for five full-day sessions.

The two-week Institute is the first experience teachers have as formal members of the SJVMP. This Institute, particularly the residential component, provides a crucible within which teachers (a) examine and reflect critically on their teaching practice and their beliefs about teaching and learning (journal writing is an important element in facilitating reflection during the week), (b) gain a deeper understanding of mathematics concepts and how those concepts span the K-12 spectrum, (c) learn new approaches and techniques for helping their students develop a deep understanding of mathematics concepts and procedures, and (d) form close, supportive collegial relationships. It is an intense week that a participant referred to as "total math immersion."

It is also an opportunity for members of the SJVMP staff to hone their leadership skills. The Institute is traditionally coordinated by the Project Director and planned in detail with the Project staff (3 faculty and 16 teacher leaders). Staff members who have participated in the planning and delivery of previous institutes serve as mentors for any new staff members, teaming with these new leaders to plan and deliver sessions at the Institute. Each staff member is expected to actively participate in at least three days of the Institute. Pooling resources, information, ideas, experiences, and expertise during the planning of the Institute and follow-up activities is an excellent professional development opportunity for both the university faculty and the teachers on the staff.

RESIDENTIAL COMPONENT: The residential week is held at a retreat center near Yosemite National Park from noon on a Sunday in June or July through the following Friday afternoon. Residential institute days typically stretch from 8:00 a.m. until 9:00 p.m., with three one-hour breaks for meals, reflection, and walks. The retreat setting allows participants a rare opportunity to "get away from day to day responsibilities and 'chaos' and focus on mathematics." A variety of experiences are provided to instruct, challenge, and support participants.

MORNING K-12 SESSIONS: During the mornings, participants work in heterogeneous grade-level groups facilitated by SJVMP staff teams. They pore over professional documents such as *Everybody Counts* (National Research Council, 1989), the NCTM *Standards* documents (National Council of Teachers of Mathematics, 1989, 1991), the *Mathematics Content Standards for California Public Schools* Issues are raised, discussed, and debated. Equity and access concerns are infused into these discussions.

During the morning sessions, participants also experience mathematics as learners and as problem solvers in a collaborative setting. Professors pose open-ended problems and investigations that serve to illustrate how numerous mathematics concepts can be embedded in the exploration, solution, and extension of a seemingly simple problem such as "How many squares can be formed on your geoboard?" or "How can you determine the height of one of the redwoods in this grove, and approximately how many cabins could you construct from it?" In debriefing, participants are amazed at the variety of solution strategies (and sometimes unique interpretations) presented by their peers, as well as by the number of mathematics skills and concepts that can be embedded in one problem. In addition, working in heterogeneous K-12 groups allows participants to (a) acquire a broader perspective of common issues and challenges that permeate all levels of education, (b) develop a greater understanding of and respect for teachers at all grade levels, and (c) become knowledgeable of how mathematics concepts are developed across the grade levels, as evidenced by one 9th grade teacher's comments:

"One of the most amazing realizations I had this week was how factors and products could be demonstrated visually/physically using base ten blocks (for whole number products) and algebra tiles (for products of variables). I had tears in my eyes as I finally understood—could SEE—the relationship between partial products in the multiplication algorithm and 'FOIL'! The fourth-grade teacher at my table and I felt a new connection in what we were teaching students…5 years apart!"

AFTERNOON GRADE-LEVEL SESSIONS: At least four hours each afternoon are spent in "Grade Level [K-2, 3-5, 6-8, and 9-12] Homes" where teachers can focus on the mathematics content and pedagogy for their particular grade level. The pair of lead teachers (SJVMP staff) who coordinate each "home" create an inviting environment in a break-out room equipped with tables, chairs, a video player, and white boards. Each room is filled with a plethora of professional books, curriculum units, manipulatives, and student projects. The Grade Level Homes provide participants with the opportunity to dialogue in depth with other teachers at their grade level and to engage in a wide variety of hands-on activities to gain a deeper understanding of mathematics concepts and ways of teaching for conceptual understanding.

In the Grade Level Home, model lessons are taught by one of the lead teachers and debriefed by the other lead teacher who demonstrates peer coaching techniques. Participants then practice teaching the concept to a partner, who critiques the lesson. Mathematics concepts that are taught at participants' grade levels (and 2 years beyond) are explored in depth. Effective uses of manipulatives and technology are demonstrated and practiced. Participants discuss honestly the challenges they face in the classroom and brainstorm solutions. Videotaped case studies are viewed and children's mathematical understanding is analyzed. Participants share "best practices." The "sharing of ideas with committed, talented teachers from many different districts" is consistently mentioned in journals as being one of the most valuable aspects of the Project, as is the opportunity to tackle challenging mathematics problems in a "safe" environment where risk-taking and grappling with concepts is expected and supported.

Darling-Hammond (1998) has stated, "teachers need deep and flexible knowledge of subject matter, many representations of ideas, and strategies for connecting ideas" (p. 5). The Grade Level Homes are an effective place to build these competencies. A fifth-grade teacher wrote, "I never had any idea what division of fractions meant or why you inverted and multiplied. It didn't make any sense that you could get a whole number from dividing fractions. Seeing how many sixths made up a half using the pattern blocks for $1/2 \div 1/6$ was the first of a bunch of big a-ha's this week! I even called my wife and told her about it. Math does make sense—you just have to know the language and actually <u>own</u> the concept, not simply memorize the procedure. I'm much more confident in my ability to use my common sense to figure out math problems or even re-derive formulas. What's that word? I'm mathematically 'empowered'!"

EVENING ACTIVITIES: After dinner, participants spend approximately two hours in their Grade Level Homes, critically analyzing a variety of mathematics curricula and aligning their own textbooks and activities with the California mathematics content standards. This exercise helps participants to be aware of and focus on the mathematics concepts contained in the activities they utilize in their classes, thus helping to avoid "activities for activities' sake." Learning to effectively debrief lessons with their students through probing questions is also an important component of the participants' leadership development experience.

After the evening sessions, participants gather to watch math-related videos such as *Good Morning, Miss Toliver* (FASE, 1993), *Mathematics–What are You Teaching My Child?* (Scholastic, 1994), and *The Story of Pi* (and others from the Project MATHEMATICS! series: http://www.projmath.caltech.edu/); to play games such as Set, Mancala, or Tri-Ominoes; to read; or to share unique skills and abilities during "talent night." Some gather for tutorials on the use of their laptops (on loan from the California Technology

Assistance Project) or to surf the Web. Most engage in conversations about teaching that typically run late into the evening.

COMMUNITY BUILDING: An important element of effective professional development is the fostering of a close-knit, supportive learning community. In such an environment, trust is developed and risk-taking is encouraged (Loucks-Horsley, et al., 1998). Fullan and Nargreaves (1998) refer to the importance of "interactive professionalism" in their essay on teacher change. The Summer Institute lays the groundwork for fostering a community where unanimity of purpose exists–the development of a deeper personal understanding of mathematics, a belief in the value of developing mathematical power in all students; and a desire to take more of a leadership role in mathematics education. The social networking of the teachers during sessions, meals, hikes, and late-night gatherings in the cabins fosters a deep sense of mutual respect and concern. Many strong, lasting professional friendships have been forged through late-night conversations between roommates and among those venturing out to the main meeting room during the late evening hours for fellowship and discussions over math games or videos. One participant referred with fondness to her "inspirational professional support group" in her post-Institute assessment.

A variety of more structured activities to develop a sense of community are incorporated into the first week. Each participant's photo is glued to a piece of colored construction paper and mounted on the wall in one of the meeting areas. This "class quilt" serves as a message board where comments are surreptitiously left for "Classmates." Each person has a secret "Math-Pal" and plans small surprises for him or her during the week. "Math Songs" such as "How do you solve a problem like division?" (a take-off on "How do you solve a problem like Maria?") are sung during meals. A class tee-shirt is designed each year by a participant-selected committee, ordered, and delivered on the last day of the Institute. (These shirts are often worn at follow-up activities and at mathematics conferences.) Engraved name badges with magnetic backs plus pens, note paper, and water bottles containing the SJVMP logo are provided for each participant as tangible mementos of the Summer Institute and membership in the SJVMP "family."

NON-RESIDENTIAL COMPONENT: The second week of the Summer Institute builds upon the first. From 8:00 a.m. until 5:00 p.m. daily, participants expand their understanding of mathematics and effective teaching strategies. In addition, recognized leaders in mathematics education and SJVMP teacher leaders give presentations on a variety of topics selected to expand the professional knowledge base and professional contacts of the new SJVMP participants. These topics include the following: , becoming an informed consumer of educational research, developing successful grant proposals, professional organizations and online journals, writing for publication, analyzing the components of the new state assessment system, the political landscape of K-12 math education, organizing extracurricular math programs, and opportunities for professional involvement.

Participants spend several hours in a computer lab each day learning ways to infuse technology into their curriculum and becoming familiar with Web sites important to their ongoing development as leaders in K-12 mathematics education. They learn presentation programs such as Microsoft PowerPoint and create an electronic presentation for their school site faculty about an aspect of their Summer Institute experience. Whenever possible, participants access electronic versions of documents and resource materials rather than print versions. For example, each of the three grade-level TIMSS reports, *Pursuing Excellence*, are available online *nces.ed.gov/timss/.* Participants delve into the report closest to their instructional grade level and discuss the findings in heterogeneous grade level groups. This sets the stage for their viewing and analyzing vignettes of Japanese, U.S. and German classrooms from the TIMSS video project.

School sites are expected to provide participants with Internet access so they can continue professional networking and can access current information. Participants are also urged to join professional associations and to have their schools become institutional members of NCTM. To help these emerging teacher leaders and workshop coordinators develop their presentation skills, they receive and discuss books such as *Sharing Your Good Ideas*: *A Workshop Facilitator's Handbook* (Sharp, 1993). They practice presentations that they plan to give at their school sites on some aspect of mathematics education. In short, they prepare for their emerging roles as mathematics education leaders.

ASSESSMENT OF THE SUMMER INSTITUTE EXPERIENCE: At the conclusion of the two weeks, participants often express enthusiasm about their Summer Institute experience:

> *"I'm exhausted, but I don't want to go. This has been a powerful experience, both professionally and personally. What a stimulating group! I have made many new friends whom I rightly respect. I have packed my brain with hours of information."*

> *"It was energizing to be around so many people who were excited about mathematics. I see myself more as a professional with new responsibilities to my students and my staff."*

> *"I feel that every session and experience was a vital necessity for improving math instruction in schools. We received some powerful messages from this project and my somewhat passive attitude has been transformed into an advocate for equitable, challenging instruction in mathematics."*

They also make comments about the intensity of the experience:

> *"These last two weeks [at Math Camp] have been one of the best years of my life!"*

Despite the intensity of the Summer Institute (perhaps *because* of this intensity), participants experience major shifts in their beliefs about the nature of mathematics, about themselves as learners of mathematics, and about what it means to teach mathematics. Typical comments are "This is the best professional development experience I have ever had" and "Many people have said that the Math Project has changed their lives—now I know what they mean!"

A SECONDARY SCHOOL TEACHER WROTE IN HER JOURNAL:

> *"Before I came to the project, I believed that I truly needed to be the 'sage on the stage' and had to be in control of every aspect of the math lesson. I was the expert, knowing and dispensing the most efficient way to solve every problem and to learn every concept. I expected all of my students to understand it the way I did and at my pace. During 'Math Camp,' I realized that I was losing so much by not listening to and learning from the students—how we all have such different ways of perceiving/solving problems and learning math concepts. Math is no longer a cut and dry subject for me. It can and should actually make sense! Students should develop confidence in the 'whys' and 'whens,' not just the 'hows' and 'whats' of math. I can't wait to teach—and learn!--with new eyes and ears!"*

AN ELEMENTARY SCHOOL TEACHER WROTE IN THE JOURNAL:

> *"I wonder where I would be now if I hadn't felt that I was so dumb in math. Math Camp was the first time that I had experienced hands-on, minds-on learning, not just regurgitation, 'fill and drill' math. I feel so much more confident in my ability to understand and teach math. I also know that I'll never be tied to an adopted math program because I now have so many resources, including my own creative ideas. The SJVMP also gave me the confidence to pursue a master's degree in mathematics education."*

As the passages above indicate, the Summer Institute experience is highly effective in increasing teacher efficacy. Participants leave the Institute with a sense of renewed confidence in their ability to reach all students and a conviction about their ability to make sense of challenging mathematics problems. Castle and Aichele (1994) discuss the importance of teachers developing professional autonomy, a key characteristic of a reflective, professional educator and of a teacher leader. Autonomous teachers are independent, confident, informed decision makers who look for opportunities to keep current professionally and who continually construct and reconstruct what they know about teaching and learning. Their views are not necessarily in alignment with or influenced by policy decisions (Firestone & Pennell, 1997). This characteristic is closely related to the concept of "self-sustaining, generative change"–the continued growth and problem solving of reflective teachers who make "changes in their basic epistemological perspectives, their knowledge of what it means to learn, as well as their conceptions of classroom practice" (Franke, Carpenter, Fennema, Ansell, & Behrend, 1998).

Although most teachers expressed a major shift in their sense of efficacy, professional autonomy, and beliefs about the nature of mathematics and teaching mathematics, some wrote that the Summer Institute experience was of value to them because it was a validation of what they already believe and how they conduct their classes. A finalist for the Presidential Award for Excellence in Mathematics Teaching wrote:

> *"For the first time in my 17 years of teaching, I can say that I feel like a professional, not isolated in my own classroom. The greatest gift the Math Project has given me is the ability to empower myself and others. For me, it came in the form of self-belief. Once I saw that my alternative teaching approaches were OK, I began to step out and become a "teacher leader." I felt comfortable sharing ideas with others, at all grade levels."*

In short, the SJVMP Summer Institute helps to develop the leadership potential of participants by providing them with an experience that

- challenges their perceptions about the nature of mathematics and what it means to be a learner of and effective teacher of mathematics
- expects them to grapple with engaging mathematics problems and with knotty issues related to mathematics education together with skilled teachers from other school sites and districts
- helps participants to develop a K-12 perspective on mathematics education issues and mathematics content
- equips them with knowledge of the latest developments in mathematics education (curriculum, assessment, instruction, legislation, studies, publications, etc.) on the state and national levels,
- deepens their understanding of the mathematics taught at their instructional level(s)
- broadens their repertoire of effective teaching strategies and resources
- strengthens their presentation skills (including their familiarity with presentation software), as well as their leadership and mentoring/coaching skills
- presents them with opportunities and encouragement for becoming involved professionally
- provides a "professional home" – a close-knit, trusted network of colleagues and friends who can provide support, feedback, perspective, and ideas

The next step in the teachers' leadership development is their participation in academic year follow-up experiences and in a Summer Leadership Institute. Each participant is also placed on an electronic mailing list for daily updates on issues related to mathematics education.

SUMMER LEADERSHIP INSTITUTE: Judith Warren Little has stated that "one test of teachers' professional development is its capacity to equip teachers individually and collectively to act as shapers, promoters, and well-informed critics of reforms" (Little, 1993, p. 130).

The SJVMP Leadership Institute builds upon the initial SJVMP experience to help achieve this goal. During the middle of the first week of the initial Summer Institute, interested alumni from past institutes attend a four-day residential Leadership Institute at the same retreat center as the Summer Institute. During this Institute, participants

- delve more deeply into the research on mathematics learning
- strengthen their mathematical content knowledge
- receive copies of and critically analyze recent articles and documents (e.g.,NCTM Yearbooks, NCTM's Principles and Standards for School Mathematics–standards-e.nctm.org, and state mathematics standards and frameworks)
- read sections from books such as Stanislas Dehaene's The Number Sense (1997) and Marilyn Burns' Math: Facing an American Phobia (1998)
- share their successes and challenges
- continue the professional networking that is an important component of the SJVMP

A statewide Presidential Award winner wrote that the SJVMP is "a network of professionals who support and raise each other to new heights." A district-wide Teacher of the Year wrote:

> *"One truly valuable aspect of being involved with SJVMP is the on-going networking, communicating, and exchanging of ideas with other teacher leaders who aren't just complaining about the way things are, but are building a vision of how things can be."*

The Leadership Retreat kicks off with a welcoming reception and a thematic party incorporating mathematics activities and investigations to give the alumni and new participants a chance to mingle. Then the alumni are given the opportunity to share what participation in the SJVMP has meant to them on a personal and a professional level. The next day, a block of time is allocated for selected alumni to give presentations to new participants, who typically view the alumni as teacher leader role models and "big brothers/sisters in the SJVMP family." A participant wrote, "It was encouraging and inspiring to hear from the alumni and know that SJVMP isn't just for 2 weeks – it's for the rest of your life!"

ACADEMIC YEAR SUPPORT: Academic year follow-up activities and reunions, as well as e-mail conversations, allow collegial relationships to deepen and strengthen over time. Participants know that they have numerous trusted colleagues upon whom they can call for ideas, advice, information, or commiseration. Wertheimer's adage that "the whole is greater than the sum of the parts" is very much in evidence when talented teachers have the opportunity to brainstorm creative approaches to challenging problems or to develop ideas for conferences or workshops. It is therefore important to give teachers numerous opportunities to reconnect with members of the SJVMP "family" in both informal settings and in more formal conference/ workshop-type settings. Each gathering becomes an opportunity to reconnect, become professionally energized, gain more knowledge about mathematics and mathematics education, and grow as a teacher leader.

Follow-up Meetings and Communication: The Project's Coordinator of Professional Development oversees the delivery of a fall "Super Saturday Sharing Session" where participants from all SJVMP classes meet together throughout the day to share and discuss student work; describe new instructional or assessment strategies they have recently employed; deepen their mathematics content knowledge; and examine new research, mathematics education documents, education policies, and educational opportunities. Three additional Saturday workshops focus on curriculum alignment with the state's mathematics content standards and ways to help students meet challenging standards. Teachers, especially those who have been in the Project for a number of years, believe that an important component of the workshops is an examination of and discussion of children's thinking, reasoning, and understanding, which is consistent with the research of Gabriele, Joram, Trafton, Thiessen, Rathmell, and Leutzinger (1999).

A variety of locally-based follow-up experiences are provided by three Regional Coordinators (RCs) and each of their four Grade Level Leaders (GLLs), all of whom have been selected by the Project Director and Principal Investigators from a pool of SJVMP applicants. In addition to their roles in planning and leading the Summer Institute, each RC and his or her GLLs are responsible for the "care and feeding" of the SJVMP teachers in one or two counties in the SJVMP service area. The Project Director supports the teacher leaders (provides financial resources from the Project budget, advice, needed facilities, etc.) and meets with the teachers on at least a monthly basis. E-mail serves as an important avenue for communication among the leaders as they plan and coordinate meaningful professional development activities for SJVMP participants and for other teachers in their region.

Each Grade Level Leader is responsible for communicating via telephone and e-mail with all of the SJVMP participants in their designated county or counties who teach in one of the four grade-level spans: K-2, 3-5, 6-8, or 9-12. The GLLs offer support and resources, collect any updates on teaching assignments or addresses for the Project's database, and serve to connect participants to the SJVMP "family." The GLL also solicits ideas for needed inservices, and is responsible for planning and delivering at least three half-day mathematics workshops a year, specially targeted for teachers in that grade-level span. These workshops are typically led by SJVMP alumni who are eager for the opportunity to develop further their presentation and leadership skills.

Each Regional Coordinator is responsible for (a) planning a full-Project reunion (consisting of a reception, meal, mathematics program, and professional updates) at a professional conference (e.g., the California Mathematics Council's Central Section conference or the Bakersfield Mathematics, Science, and Technology Conference); (b) coordinating and supporting the activities of the GLLs in his or her region, and (c) hosting an annual Administrator Evening event for SJVMP participants and their guests (principal, superintendent, and/or school board member). At these dinners, administrators are given the chance to learn more about the SJVMP and Project-related experiences of their teachers, to recognize the contributions of the SJVMP participants at their school sites, to receive updates regarding mathematics education (e.g., curriculum, assessment, pending legislation, national trends, and research), to solve mathematics problems along with their teachers, and to be recognized for their support of the Project and their teachers' leadership efforts. Each administrator leaves with a file containing information about the SJVMP, regional mathematics education opportunities, and salient articles or resources. Interested administrators also receive a computer disk containing a searchable 60-field "Professional Development Providers" database containing the names of all SJVMP teachers who are interested in delivering assistance to school sites or districts on a wide variety of math-related topics. Finally, administrators are given the opportunity to be added to the Project's e-mail distribution list to receive articles and news regarding mathematics education.

The role of the principal and other key administrators is vital "for professional development to move from learning to changes in classroom practice"(Loucks-Horsley, Hewson, Love, & Stiles, 1997), as well as for the support of leadership activities. In addition to the Administrator Evening activities, letters from the Project Director and Principal Investigators to principals thanking them for their support and requesting feedback are mailed periodically.

ONLINE NEWSLETTER: For ongoing professional growth, it is also vitally important that members be kept apprised of the latest information and opportunities related to mathematics education. Fullen and Hargreaves (1998) write, "redefining the teacher's role includes a responsibility to become knowledgeable about policy, and about professional and research issues in the wider state, provincial, national, and international arenas." For a number of years, the SJVMP newsletter, "Summing Up," was a key source of information for members of the mathematics education community. To provide a much more cost-efficient and timely way to share pertinent information, the SJVMP mathematics education e-mail network was born. The Project Director collects, annotates, and disseminates via e-mail articles and announcements pertaining to mathematics education, URLs of newspaper and journal articles, details of professional development opportunities, and Project member updates on a daily basis. Over half of the SJVMP teachers are part of this network, and over 400 more individuals (including curriculum coordinators, consortia directors, superintendents, professors, project directors, and interested individuals from the national and international mathematics education community) are on the expanded distribution list and receive a subset of the messages sent to SJVMP participants. Feedback from participants (and their administrators) has been highly supportive of this online newsletter, which helps to keep readers on the cutting edge of developments and leadership opportunities in mathematics education.

> *"The email hotline is an incredible resource for me. The news and articles keep me amazingly well-informed. I forward the messages to key administrators in my district and also print them out and put them in a notebook that I leave in the teacher's lounge."*

> *"Knowing what's happening in the wider educational arena gives me a more authoritative voice and chance to help shape the vision of the district in which I work. Being in touch with the leading edge of educational research, teaching innovation, and current events has given me the confidence to share my expertise and viewpoint in a professional way, and it has gained for me the respect and validation of my colleagues and administrators."*

EVIDENCE OF TEACHER LEADERSHIP: SJVMP participants have enthusiastically commented that the Project has given them an exciting new vision as well as a deeper understanding of mathematics content and its instruction. Teachers are implementing innovative programs of mathematics instruction in their classes and in their schools. In addition to opening participants' eyes to the world of mathematics reform, the Project is credited with providing a wide array of opportunities for professional growth. Numerous leaders in California's Central Valley credit their leadership positions and professional recognition to their participation in the SJVMP.

> *"Before the Math Project, I was a really good elementary school teacher, bordering on excellent, experimenting on my own with manipulatives, problem solving, etc. But I was on my own. Very little information on good resources. Clueless about the fact that there were other teachers on the same road. Basically minding my own business. Hiding from administrators who might object to my methods. Afterwards, wow! I was a better teacher, but I also started bugging administrators for more money for math resources, leaving copies of articles and great activities in teachers' mailboxes, doing mini-in-services at teachers' meetings, applying for and getting mentorships, getting on curriculum committees and math textbook adoption committees. Because of my increased visibility and involvement, I was one of only three California teachers to be selected by NASA to attend a 2-week workshops at their Jet Propulsion Lab in Pasadena and was also named district Teacher of the Year."*

> *"Before I became a member of the SJVMP, I was considered a strong math teacher, but I was isolated. The classroom was as far as my professional life went. The Math Project opened my eyes to a plethora of opportunities and opened dozens of doors for me! My world has expanded dramatically! My life is full of new challenges, avenues for professional growth, and dozens of new professional colleagues and friends. I guess you could say I am "professionally self-actualized"!*

"My SJVMP leadership training inspired me to do many more activities and unit Projects with my students; share them with other teachers at conferences; share them with my math department (which led to me being elected Department Chair at both schools at which I've taught); share them with thousands of math teachers through the state math journal, the ComMuniCator; share my time by being on statewide math committees, and so on."

Administrators have been highly supportive and enthusiastic about the effect of Project participation. Teachers are expected to take a wide variety of leadership roles at their school site and in their districts. Administrator surveys collected by Bohlin (1996) included the following comments:

"Every teacher comes out stronger and more proficient in curriculum and assessment. The experiences empower teachers to make significant changes in their classrooms and schools."

"As Director of Curriculum, I have seen four of our teachers participate in the project. They have all served in leadership roles including mentorships, math committees, presentations, Program Quality Review for math, and officers in math organizations."

"C--- has gained confidence in his field. He is willing to share his knowledge with others and to move the department in the direction of the frameworks. Student interest is high."

"I think that this project is wonderful. I hope to send others who will take risks in the classroom and share with others."

"M---'s leadership is changing the way math instruction is delivered at this school. He is an excellent teacher who is now a teacher of teachers."

"It's been a pleasure to work closely with a teacher who is now on the 'cutting edge' of mathematics instruction."

Following is a sample of recent leadership roles and professional recognitions that teachers have received in the years following their initial participation in the SJVMP:

- Four SJVMP teacher leaders have received the statewide Presidential Award for Excellence in [Elementary or Secondary] Mathematics Teaching during the past three years; three more have been finalists. Two teachers received district Teacher of the Year honors in 1999.
- A team of a dozen SJVMP teachers from Visalia Unified School District was instrumental in conceptualizing an NSF Local Systemic Change grant that was recently funded for $3.8 million; four members of the development team are

currently serving as full time lead teachers for that project.

- SJVMP teacher leaders serve in key leadership roles on the mathematics inservice team for Fresno Unified School District's (FUSD's) NSF-funded Urban Systemic Initiative; an SJVMP teacher was selected as the district's Director of Research and Evaluation.
- A team of SJVMP teacher leaders co-directed a statewide conference on equity in mathematics education.
- Three NCTM regional affiliates have been established or revitalized by SJVMP teachers.
- The immediate past-president of the California Mathematics Council (CMC) is a former co-director of the SJVMP; a number of other SJVMP teacher leaders have been elected to serve on the CMC statewide or regional boards in recent years.
- Three SJVMP teacher leaders' classrooms were filmed for the WGBH (Boston) Math Library Project.
- Over 20 SJVMP teachers have written Internet-based mathematics lessons linked to the state mathematics content standards for SCORE-Math (http://score.kings.k12.ca.us/lessons.html) and/or have written mathematics lessons to accompany instructional television (ITV) programs. Teachers involved in writing the ITV lessons provided training for other teachers at the KQED (San Francisco-based) National Math, Science, and Technology Conference in 1994.
- Three SJVMP teachers were selected as mathematics resource teachers, coordinators, and writers for a statewide online high school program, CyberHigh.
- Over 50 teachers were trained in the use of replacement unit curricula and delivered regional inservices
- Ten to twelve SJVMP teachers serve as part-time instructors each semester for mathematics or mathematics methods courses for preservice teachers at area colleges and universities.
- Over 200 of the SJVMP participants have served as school site mentor teachers (math specialization), mathematics workshop leaders, and/or presenters as local and regional mathematics conferences
- Three SJVMP staff members were recently selected to be the Fresno and Tulare County Mathematics Coordinators and the regional coordinator of the California Technology Assistance Project
- In the past several years, members of the SJVMP have served in a number of leadership roles at the state level, including service on the California Framework Commission, statewide mathematics assessment and accreditation panels, and state mathematics curriculum advisory panels.

POSTSCRIPT: The above profile of the San Joaquin Valley Mathematics Project provides but one example of the effectiveness of the California Mathematics Project model in developing K-12 mathematics teacher leaders. Participants enthusiastically credit the Project with having prepared them for and supported them in their leadership roles. For many years, the vision of the CMP, site directors, staff, and teachers was consistent with the state's vision of progressive educational reform. In 1986, then-State Superintendent of Public Instruction Bill Honig observed, "There is a growing spirit of optimism and a can-do attitude among educators that I think bodes well for the future of the state and the future of the country." For members of the CMP community, this spirit of optimism has been sorely tested recently, affecting the community's strength, effectiveness, productivity, and morale.

In recent years, California has experienced a powerful, effective backlash against the NCTM *Standards*, the reform-oriented *Mathematics Framework* (CDE, 1992), and reform-oriented mathematics curricula (e.g*., MathLand, Connected Mathematics,* and the *Interactive Mathematics Program*) by individuals and groups with important political ties. For reviews and commentary, see Becker and Jacob (1998), Carlos and Kirst (1997), Jackson (1997a, 1997b), and Jacob and Akers (1999). These writers document how policy decisions regarding the *Mathematics Framework* (1999), *Mathematics Content Standards* (http://www.cde.ca.gov/board/mcs_intro.html), textbooks, and assessment were taken out of the hands of teacher leaders and mathematics educators and placed primarily in the hands of selected mathematicians and members of the State Board of Education (SBE). The SBE recently approved a new *Mathematics Framework* with a more traditional focus and containing rigorous, skills-based content standards on which a new assessment system, STAR (http://star.cde.ca.gov/) is based. A recent mathematics textbook adoption (AB 2519) resulted in the approval of a limited number of traditional books that were closely aligned with the state standards but considerably different from recent textbook analyses/recommendations from national groups such as the American Association for the Advancement of Science.

In early 1998, Governor Pete Wilson removed the California Subject Matter Projects from a draft version of his budget. Negotiations restored the projects, but under new legislation – AB 1734 (Mazzoni). This bill requires the CSMPs to use the state content standards as the principal reference point for CSMP-sponsored professional development activities. Mathematics content is to be the primary focus of California Mathematics Project institutes and follow-ups. CMP sites must form partnerships with low-achieving schools and provide mathematics instruction for these schools' teachers. The test performance of students in the partnership schools is to be monitored and used as one of each project site's accountability measures.

Following the introduction of AB 1734, CMP sites were required to re-conceptualize their programs and submit proposals in accordance with the new legislation. Only 11 sites out of the 17 that had been in operation for nearly a decade received funding for 1998-1999. Fifteen of the 19 sites that submitted proposals in 1999 (13 of the original sites) were funded for 1999-2000. In late 1998, the entire staff of the Executive Office of the CMP resigned, and as of August, 1999, a permanent Executive Director had not yet been named.

The next few years will be a time of reflection, reconceptualization, and rebuilding for the California Mathematics Project. The former CMP model embraced and built upon the tenets of a progressive mathematics reform movement, which was supported by and consistent with existing statewide policies for curriculum, instruction, and assessment outlined in the opening pages of this chapter. Teacher leaders (and teacher educators) were actively involved in curriculum development and assessment; they were part of the decision-making process, an important element of teacher leadership.

The new CSMP model, under AB 1734, is consistent with the current statewide focus—one that does not embrace the tenets of the reform movement, but instead focuses on a more traditional skill development/assessment model where teachers' professional involvement and decision-making opportunities are more restricted. The former CMP model was highly successful in developing a thriving statewide cadre of enthusiastic, knowledgeable, professionally-involved teacher leaders for K-12 mathematics. The CMP also provided a productive professional network for university mathematicians and mathematics educators who were involved as Project Directors and Principal Investigators. The capacity of the "new CMP" to recapture the momentum and success enjoyed by the "old CMP" is difficult to predict. The final story of the California Mathematics Project will be a revealing study of how two contrasting models compare in their effectiveness in developing and sustaining K-12 mathematics teacher leaders.

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Dr. Carol Fry Bohlin is currently a Professor of Mathematics Education at California State University, Fresno and served as the Director and Principal Investigator of the San Joaquin Valley Mathematics Project from 1990 to 1999.

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THE ROAD NOT TAKEN: A CASE STUDY IN THE SEARCH FOR COMPUTATIONAL POWER AND CONCEPTUAL UNDERSTANDING

ROBERT STEIN, EMERITUS PROFESSOR OF MATHEMATICS, CSU SAN BERNARDINO

INTRODUCTION: Conceptual understanding and computational power are sometimes seen as opposing goals in mathematics education. In fact they are not mutually inconsistent and are actually reinforce each other. This paper illustrates that fact in the case of logarithms and exponents, and outlines an approach, which, though passed over by history, offers significant benefits for today's mathematics curriculum.

ON COMPUTATION: The French mathematician Chuquet, writing in 1484, juxtaposed the sequences of whole numbers and doubling numbers (today we call them powers of two) as shown below:

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	2	4	8	16	32	64	128	256	512	1024	2048	4096	8192	16384	32768

and observed that addition in the top row corresponds to multiplication in the bottom row. For example, to multiply 64 by 128, one could look at the numbers above them, 6 and 7, add those to get 13, and locate the product of 64 and 128 directly below 13.

SOME HISTORY: Chuquet was one of several scholars to notice this, but nobody explored it in depth until the late 16th century. Then John Napier (1550-1617) in Scotland and Joost (or Jobst) Bürgi (1552-1632) in Prague, used that simple observation to revolutionize computation. Both men began their work in the 1580s, but Napier published his table of logarithms in 1614, while Bürgi, despite the entreaties of his colleague Kepler, did not get around to publishing his Arithmetische and Geometrische Progress Tabulen (Arithmetic and Geometric Progression Tables) until 1620. Bürgi's delay cost him any claim to priority, and in the six years before his tables appeared, Napier's logarithms were so widely accepted, so extensively improved and extended, that Bürgi's table, when at last it appeared, was already out of date. It remains relatively unappreciated today, even though, had it been more widely recognized, mathematics might have developed very differently than it did. As you will see, this "road not taken" could be used today to teach about exponents and related topics in a way that naturally motivates students and is easy to understand.

Chuquet's observation is fine for multiplying the numbers 1, 2, 4, 8, 16, etc. by each other, but it is no help for multiplying other numbers. Bürgi's idea was to replace the numbers 1, 2, 4, 8, 16, . . . with a different geometric sequence that grows much more slowly. You could start with 1, as Chuquet had done, instead of repeatedly multiplying by 2, you could multiply by a ratio just slightly smaller than 1. The resulting sequence would grow very slowly and have numbers very close to whatever number you would like to multiply by.

MULTIPLICATION: Bürgi actually multiplied by 1.0001, but we will illustrate his idea with a ratio of 1.1. In Table 1 below, we describe the resulting numbers as powers of 1.1, but the theory of exponents was not fully developed until nearly a century after Bürgi died.

TABLE 1: POWERS OF 1.1^n

n	Approximate value of 1.1^n	n	Approximate value of 1.1^n	n	Approximate value of 1.1^n
1	1.1	18	5.5599	35	28.1024
2	1.21	19	6.1159	36	30.9127
3	1.331	20	6.7275	37	34.0039
4	1.4641	21	7.4002	38	37.4043
5	1.6105	22	8.1403	39	41.1448
6	1.7716	23	8.9543	40	45.2593
7	1.9487	24	9.8497	41	49.7852
8	2.1436	25	10.8347	42	54.7637
9	2.3579	26	11.9182	43	60.2401
10	2.5937	27	13.1100	44	66.2641
11	2.8531	28	14.4210	45	72.8905
12	3.1384	29	15.8631	46	80.1795
13	3.4523	30	17.4494	47	88.1975
14	3.7975	31	19.1943	48	97.0172
15	4.1772	32	21.1138	49	106.7190
16	4.5950	33	23.2252	50	117.3909
17	5.0545	34	25.5477		

This table can be used just like Chuquet's table. For example, to multiply 4.5950 (entry 16) by 13.1100 (entry 27), just add 16 and 27, and look up entry number 43. It is 60.2401, or 4.5950x13.1100. If you multiply 4.5950 x 13.1100 you get 60.24045, which is close to what we got from the table. The discrepancy comes from keeping only four decimal places in the table entries, as indicated by the word "approximate" in the column headings. Of course, this table has limitations. For example, to multiply 66.2641 by 23.2252, we would add the indices, 44 and 33, and get a sum of 77, but the highest index in the table is 50.

DIVISION: If multiplication can be done by adding the indices of the table, then division can be done by subtracting them. For example, 72.8905 ÷ 14.4210 can be found, at least approximately, by subtracting 28 (the index of 14.4210) from 45 (the index of 72.8905) to get 17 and looking up the corresponding entry, 5.0545. Again, it is a good exercise to calculate the quotient directly and find the error incurred in using the table. It is only a small percentage of the true answer. As before, the use of this table is limited; in this case, dividing a smaller number by a larger one leads to a negative index.

POWERS: Since powers are obtained by repeated multiplication, and multiplication corresponds to addition of indices, then powers must correspond to repeated addition, or multiplication, of indices. For example, to find 2.14365, multiply the index of 2.1436 (which is 8) by 5 and look up the 40th entry, or 45.2593. Again, it is a good exercise to compare the answer from the table with the true answer and find the error as a percentage error of the true answer.

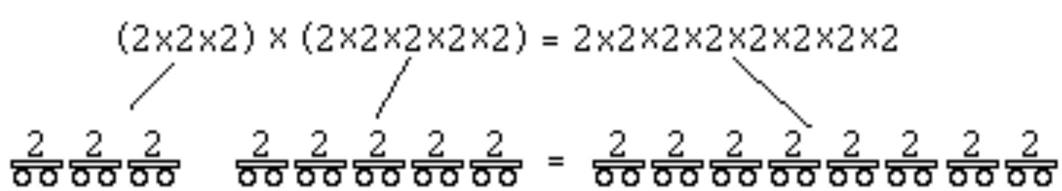
ROOTS: Roots are generally even more of a nuisance to compute than powers Square roots can be approximated with a bit of trial and error, but more sophisticated methods are needed for higher roots. However, good approximations are easy to get from our table! For example, to find the fifth root of 28.1024, find the index of 28.1024 in the table (it is 35) and divide that by 5 to get 7. The corresponding table entry, 1.9487, is, at least approximately, the fifth root of 28.1024, as you can check with a calculator. (You probably would not want to check this by doing the arithmetic yourself.) The limitation of the table computation here is that division of indices can lead to a fraction. For example, the square root of 15.8631 would be found by halving its index, 29, and looking up the entry corresponding to 14.5, but our table does not have fractional indices. A natural thing to try here is to associate the 14.5 step with the average of the 14th and 15th steps, which is 3.98775. Since the actual square root is about 3.98285, this idea appears to merit further investigation.

A LITTLE THEORY: Bürgi developed his table without ever mentioning exponents, but they are the key to understanding the table and overcoming its limitations. The key to all such tables is summed up in the following example based on Chuquet's table.

To multiply 23 by 25, think of 23 and 25 as trains of 2s, one with three cars, the other with five. Hooking the trains together (multiplying the powers) forms a single train, which of course has 3 + 5 = 8 cars, as in Figure 1.

FIGURE 1: HOOKING TRAINS OF NUMBERS
TWO TWO-TRAINS MAKE ONE BIG TWO-TRAIN

$$2^3 \times 2^5 = 2^{3+5} = 2^8$$



This observation holds not only for trains (powers) of 2s but indeed for trains of any number. Thus our example of multiplying 4.5950 (entry 16) by 13.1100 (entry 27), involves hooking together trains of 1.1s, one with 16 cars and the other with 27, to make one 43-car train. In terms of exponents,

$$\mathbf{4.5950 \times 13.1100 = 1.1^{16} \times 1.1^{27} = 1.1^{(16+27)} = 1.1^{43} = 60.2401}$$

Since powers are obtained by repeated multiplication, they may be seen as formed of multiple copies of the same train. For example, to find 3.13843, first consider 3.1384 as 1.112. Then 3.13843 is the cube of 1.112, which can be thought of as three 12-car 1.1 trains hooked together to make a 36-car 1.1 train. Then 3.13843 = 1.136, which we can look up in the table as 30.9127.

LAWS OF EXPONENTS: In modern notation for exponents, invented by Descartes in 1637, we have been using this basic fact: $b^m\, b^n = b^{m+n}$ for any positive number *b* and positive integers *m* and *n*. Applying this repeatedly leads to a second important fact: $(b^m)^n = b^{mn}$ for any positive number *b* and positive integers *m* and *n.*

Among the early writers who made observations similar to Chuquet's some, such as Al Samawal (1125-1180) had had extended the geometric progression to the left to get:

$$\frac{1}{128},\ \frac{1}{64},\ \frac{1}{32},\ \frac{1}{16},\ \frac{1}{8},\ \frac{1}{4},\ \frac{1}{2},\ 1,\ 2,\ 4,\ 8,\ 16,\ 32.$$

Why not continue the arithmetic progression in the same way, like this?

$$\begin{array}{ccccccccccccc} \frac{1}{128} & \frac{1}{64} & \frac{1}{32} & \frac{1}{16} & \frac{1}{8} & \frac{1}{4} & \frac{1}{2} & 1 & 2 & 4 & 8 & 16 & 32 \\ -7 & -6 & -5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 \end{array}$$

Now, if you divide 32 by 512 by subtracting exponents, you get $2^5 \div 2^9 = 2^{-4}$, and - 4 lines up with the correct answer, $\frac{1}{16}$. That makes it natural to define $b^{-n} = \frac{1}{b^n}$ for any positive number *b* and positive integer *n*!

The basic law of exponents applies regardless of the signs of the exponents:

$b^m b^n = b^{m+n}$ for any positive number *b* and integers *m* and *n*. With negative exponents available, the table in Table 1 can be applied to numbers outside its range.
For example, multiply 5,0545$\times$1331 this way:

$$\mathbf{5{,}0545 \times 1331 = 5.0545 \times 10^3 \times 1.331 \times 10^2 = 5.0545 \times 1.331 \times 10^3 \times 10^2}$$

Multiply the first two factors by the table, getting 6.7275, and apply the basic law of exponents to the powers of 10 to get $6.7275 \times 10^5 = 672{,}750$.

RADICALS: The path to fraction exponents is equally natural. If taking square roots in the table involves dividing the index by 2, and taking cube roots involves dividing the index by 3, it is natural to define $b^{1/2} = \sqrt{b}$, $b^{1/3} = \sqrt[3]{b}$, and, in general, for any positive number b, and integers m and, $b^{m/n} = \sqrt[n]{b^m}$

BÜRGI: Bürgi's actual table is over 23,000 entries long, which is too large for classroom use, but just the entries in Table 1 makes his ideas accessible to students as low middle school. Bürgi began work on his table in the 1580s, but his work was largely overlooked in the enthusiasm for logarithms. But this work by Bürgi is a natural and deserved addition to the modern exponent notation invented by Descartes in 1637 nearly a half century after Napier—and Bürgi had, independently—begun work on their tables. Wallis in 1655 hinted at the idea of rational exponents, and Newton made that idea explicit in 1679. The first comprehensive explanation of rational exponents did not appear until 1708.

The approach to exponents described in this article is not usually found in textbooks today and does not correspond to the historical development of these ideas. Nevertheless, there is much to recommend this "road not taken" for use in today's classrooms.

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UNDERSTANDING STUDENTS' PERSPECTIVES: MATHEMATICAL AUTOBIOGRAPHIES OF UNDERGRADUATES WHO ARE NOT MATH MAJORS

SHANDY HAUK, SENIOR RESEARCH FELLOW, WESTED

INTRODUCTION: A student encounters a mathematics problem. The student stares at the problem, brain working furiously. However, to an outside observer, little in the way of mathematical activity seems to be happening. It turns out that the working memory needed for thinking about a mathematical problem situation can be taken up by dealing with intrusive thoughts and emotions sparked by the interaction with mathematics. Some research suggests that expressive writing – organizing and reporting memories – may help in dealing with stress, particularly with anxiety associated with memories (Cameron & Nicholls, 1998; Pennebaker, 1993). Writing a mathematical autobiography of the sort reported on here is an expressive writing activity.

When we create a narrative from poorly organized recollections of past mathematical selves, it allows for the repackaging of partially and loosely connected memories into streamlined memory structures that may be dealt with more efficiently (Conway & Pleydell Pearce, 2000). However, the effect may not be immediate and may not be restricted to negative experiences (King, 2002). Klein (2002) reported that though improvements in working memory were small one week after an expressive writing assignment, they were statistically significant seven weeks after the assignment: the greatest increase in working memory (11%) was among students who had written about a negative experience, a smaller increase (4%) occurred for students who had written about a positive experience. No significant change in working memory was evidenced among students who had written about time-management instead of events of personal significance.

Just as undergraduates in mathematics are likely to have quite different personal mathematical histories from their college mathematics instructors, most middle and high school students have different experiences with, and expectations of, mathematical learning than their teachers. And certainly, most students will not have the same careers as their teachers. Research has noted that some teachers, particularly those still in the formative career stage (e.g., the first five years of full time teaching), may "give up too easily" when difficulties arise in communicating with students (Borko, Eisenhart, Brown, Underhill, Jones, & Agard, 1992). Part of the development of the knowledge for effectively teaching mathematics involves anticipating, and incorporating into teaching, the manifold abilities, experiences, and concerns of students.

NON-MATHEMATICS MAJORS: This article offers insight into the primary and secondary school experiences of a particular group of learners: non-mathematics majors enrolled in lower division "service" courses like college algebra, liberal arts mathematics, pre-calculus, mathematics for future school teachers, and various flavors of calculus (e.g., applied calculus for business, biology, etc.). Results are from Hauk (2005), which reports on experiences sampled from over 300 student essays. The names of all people and places have been fictionalized. Though the assignment is included as an appendix, please read the *Note on using the assignment* at the end of this article before making a decision to use it. While the particulars may be unsettling for some readers, the vignettes offered here are authentic and representative of the experiences of much of the U.S. college-going population. I thank the hundreds of undergraduates who have taken the time and energy to share their stories by completing a mathematics autobiography essay.

Finally, a suggestion to the reader: put on your student hat now and remember what it was like for you as a child or young adult learning mathematics. For example, what can you recall of learning to count? . . . learning to tell time? . . . learning what fractions mean? . . . learning how to use money?

When asked to recount their personal mathematical autobiographies, student accounts of success (and lack thereof) in mathematics range from before birth,

> *I was successful doing math in the womb - I divided from one cell into two...that was the last time I was successful in math.*

to sixth grade,

> *Locked in my bedroom I would scream at the top of my lungs, `WHO CARES ABOUT THE PROBABILITY OF GRABBING A GREEN MARBLE!!'*

to high school,

> *I was terrified of Geometry, and thought Calculus was something that I could never achieve, let alone master. Of course once I got in and tackled these subjects, I found that it was not that bad. ...Well, I have come to learn that I might actually like math, and that I am not half bad at it.*

and undergraduate service courses:

> *I was 31 years old but determined to get a college degree. I had a hard time following the program in this class [algebra at a community college]. I struggled. When I received my first test back I had gotten a D and the girl sitting next to me got an A. I thought, 'what does she have that I don't?' It was a tattoo, a pierced nose, ear, lip, eyebrow, and a Mohawk. I dropped that class. I guess I had my hands full with the kids at home, ages 3, 5, 7, and 9.*

In comparison, those with a long-time love of mathematics who have succeeded in advanced mathematical learning, including many high school teachers, tend to focus on the "thrill of that moment, when you just *know* you have it." There is a shared passion for the certainty that comes with understanding in mathematics:

> *An infatuation with learning mathematics has guided my life goals. Until I took really advanced math, I can only remember success being related to mathematics. Even though advanced course work brought my first of many encounters with failure in mathematics, I have continued to study it, and the only explanation I can give for this is this idea of "addiction" to attainment. That despite the agony of confusion, I still feel confident I will make sense of things and in that heart-pounding, breathless, spine-tingling instant of AHA, I will find all the reward I need.*

Like those with degrees in mathematics, the majority of those who identify themselves as future secondary teachers, tell positive stories about inspirational teachers and "inherent motivation – I just have to figure things out" and may remark that "I love math because it is like a puzzle, and I learned about that from logic puzzles, not in school."

By comparison, among future K-8 teachers, stories are just as likely to relate positive associations with mathematics as they are to mention negative ones. Here is how Karen, a mathematically able pre-service teacher in her final semester at university put it,

> *I always try to find the answer right away and if I can't find it, then I say those three words, "I hate Math." Sure, I'll eventually find the solution, but I'll be frustrated and upset the whole time doing so.*

Across the spectrum of learners, deep and powerfully experienced emotions – thrill, satisfaction, disappointment, agony, hate, and passion – are connected to mathematics.

THE ROLE OF THE EVIDENCE OF AUTHORITY – TEACHER AND TEXTBOOK: In their written reflections about becoming intimate with mathematics, most students referred to the powerful "authority" of the instructor and of the text. One quarter of students (equally split between men and women) discussed the idea of coming into some ownership of their understanding and learning of mathematics after coming to college. As Dan, a health sciences student, wrote:

> *Up until now [college], I've always had to just accept that what the teacher says is fact but never understood why a formula works the way it does... I've been given the knowledge on what to do, but never what to do with it. In my view, it's like giving a kid a hammer and teaching him how to swing it. But if the kid doesn't know that a hammer is used to drive nails, it's worthless to him. Math has been worthless to me until now.*

College students planning to become teachers talked about the need for the authority of the teacher and the "complexity" of mathematics most often when discussing geometry – a high school course that for many was a first exposure to proof. For Violet, who planned to teach middle school, the ultimate arbiters of truth were the teacher and textbook and it was pointless to try to convince one of those arbiters:

> *I had a hard time getting past the fact that the triangles, circles, and squares on the paper were not actually the size that was stated, and why I had to prove something that the teacher already knew was correct.*

Violet saw herself as a collector of mathematical truths and saw her future teacher-role as curator of the collection. Violet reported, as did 15% of prospective teachers, that the ultimate authority was the "teacher's edition" of the textbook. Most undergraduates are astounded to learn that the instructor in a college mathematics course may have no "teacher's edition." Several reported feeling lost and betrayed by this lack of authority. They reported having to "settle" for relying on themselves. In an interview about her mathematical autobiography after Violet completed her first college mathematics course, she expanded on the idea of the authority of the textbook:

> *You have to be on the look out, you know. … I'd always figured if you don't know, well, go look in the book, in the back of the book. This whole idea of having to think, you know, DURING class...this was very hard for me. ["Humphing" noise] Yeah, I still don't like it, I don't know...I'm starting to not even be sure I want to be a teacher, you know? I mean, all these kids are going to look at me like I'm supposed to know and, well, will I? And if I don't, I'll have go to find out, you know? That's a lot of work! [laughs]*

In other student talk about geometry, pre-service teacher Jennifer noted:

> *I'm planning on becoming an elementary teacher, but if I were to ever change over to high school, I would like to teach Algebra or geometry (please note, crossing truth tables as a teacher wouldn't be that bad because I will have the teacher's book).*

Again, the perception that a mathematics teacher is not necessarily one who is knowledgeable, but rather one who has access to external sources of information – as in the teacher's edition of a textbook, arises. Like many pre- and in-service teachers (Spangler, 1992; Tatto,1999), Jennifer's personal philosophy regarding the nature of mathematics included mathematics content and processes as fixed, algorithmic, and external, "out *there*."

While many successful mathematics teachers and graduate students credit a mentor, typically a high school teacher, with challenging, encouraging and assisting them into the pursuit of a mathematics-related career (Carlson, 1999), the non-math-major undergraduates seemed to be coming from a complementary state: 60% credited a teacher with boring, discouraging, or hampering them in mathematics. This led to what many identified as mathematics-avoiding career choices. One-fifth of this group of "math-avoider" students were prospective school teachers. However, no connection appeared to exist between a student's reported judgments of teachers as "good," "bad," or "indifferent," and that student's identifying her or himself as a "math-avoider." In fact, 20% who were prospective teachers said they valued mathematics "in spite of" a bad or indifferent teacher. They wanted to become good teachers of mathematics *because* they had experience with bad or indifferent teachers.

CHEATING: Students sometimes identified their peers as more authoritative sources than themselves. About 20% told stories of cheating in primary, middle, or secondary school as a means to gain information on exams. Tanya, a communications student, cheated early (starting in the first grade) but was not confronted about her cheating until third grade:

> *Third grade was my first time that I got caught cheating. A boy named Griff Peels was really good at math, and I used to sit right by him. A few times, when we were doing work out of [the] textbook and I wasn't understanding a problem I would look at his paper ever so casually. I thought I was being sly, but I got caught. The teacher didn't catch me, Griff caught me... At first I denied it, but eventually I told him I had. He told me that he didn't care and that I could when I needed to.*

The tendency to cheat reasserted itself in her experiences repeatedly until college, at which point she finally failed a course: "I took Math 71 [intermediate algebra] three times before I passed." Interestingly, Tanya's epiphany about the "waste of time and energy involved in cheating" happened in her third go of Math 71 when students did an activity where they shared their mathematical histories.

Heather, a 20 year-old television journalism student in her second year, started cheating in the second grade, stopped for a while, and picked it back up again in high school pre-algebra. She reported in her interview that the subjects she had cheated in were "math and science, but that's really all the same thing anyway, *math*." Heather's second grade teacher, Mrs. Forth, discovered her copying from a fellow student, Jackie, and seated her away from that student for the next exam:

> *I remember getting the test back as if it were yesterday. I received no scratch and sniff sticker no gold star just a big "F" in red magic marker. Well there was proof and I was found a cheater in the court of second grade. I then had to spend every recess for two weeks with Jackie Town who I hated more and more by the day listening to her babble on about the rules of subtraction (why she was punished is beyond me). There are two things I learned from this, subtraction, ...and also, if I am going to cheat I might want to change up my answers a little bit from the person I am cheating off of.*

Heather returned to cheating in high school when she went from public school to a private school. She remembered the lesson from second grade and reported, "My classmates and I had discovered the art of cheating and practiced it ritualistically." She recalled in her interview "successfully cheating my way to a B in Sister Ruth's class . . . Sister never did catch on yet I realize now that I was only cheating myself. The SAT's proved that . . . "

Roberta, a prospective teacher who failed the first exam in her high school algebra class, was not proud of her one and only foray into cheating:

> *Well, instead of going to my teacher and asking for help, me and my girlfriends decided that I should cheat off of them, so I did. Looking back I have no idea how we were never caught cheating, but we weren't. I remember being nervous and having major anxiety everyday during math class. I also remember that I managed to pass algebra 1 with a B, but I would definitely change this memory if I could.*

In a later interview, Roberta referred to her anxiety about mathematics repeatedly and said she felt it all went back to the fact that algebra felt "impossible" to her.

One student discussed cheating for a distinctly different reason. George, a communications student, talked about his sixth grade basic mathematics course where homework was graded on the honor system – the teacher read answers out loud, students scored their own homework and then verbally reported their score when roll was called.

> *When I got a tutor I did not want my friends to know because I thought they would think of me as being stupid [for needing help]. All of a sudden my homework grades were much better but when the teacher asked us what we had scored on the homework I would give her a lower grade than I had really received thinking that she would say I was cheating, or not grading my paper correctly.*

It would appear that some of George's mathematics self-evaluative habits were already fairly well formed by age 12. Despite knowing he was doing as well as or better than his peers, he felt he could not take the risk of exposing his improved understanding of mathematics.

Fewer than 10% reported "copying homework" of friends. This form of cheating was seen as quite different from cheating on a test. Getting a test answer from someone else was a screen for a lack of understanding. However, homework copying was seen as a way to satisfy a behavioral demand of an external authority (the teacher): "homework usually didn't have anything to do with really *learning* anything anyway." The tale told by Marcus, a first-year music student, epitomizes the statements by those who reported homework copying. Marcus moved to a new school in mid-term and had to "catch up":

> *Let's be real; what would any high school student do in this position? Copy. Of*

course. There was no other way for me to succeed. I had all of my homework, flunked the few make up tests, and by the time I caught up and got the hang of things the semester was almost over. The "As" that I earned at the end of the semester balanced out the "Fs" that I earned at the beginning. The result was a "D." So much for being valedictorian.

GRADES: Virtually everyone talked about grades as an aspect of their relationship with mathematics. In fact, in the hundreds of essays the author has read (from my own students and those of others), in only one essay was there no mention of grades. Moreover, in the social contract that underpins schooling in the United States, teachers may have access to a great deal of power through grading and grades. Student reports reflected their experiences with the many ways that people assert and wield power.

Students frequently used forms of the word "get" when discussing mathematics grades. After student interview responses had clarified some usages, context analysis was used (cautiously) to determine whether the grade "got" by a student was perceived as *earned* (some attribution of self-responsibility for, or ownership of, learning) or *received* (some attribution of external responsibility for learning).

Among women, 80% reported "receiving" grades with 34% "earning" them (some reported both). Among the men, 73% reported "receiving" grades while 64% stated they had "earned" them (again, some reported both). Figures 1 and 2 give the frequency of each type of grade in each category by sex. Every person who "earned" an A or B also reported having "received" an A or B.

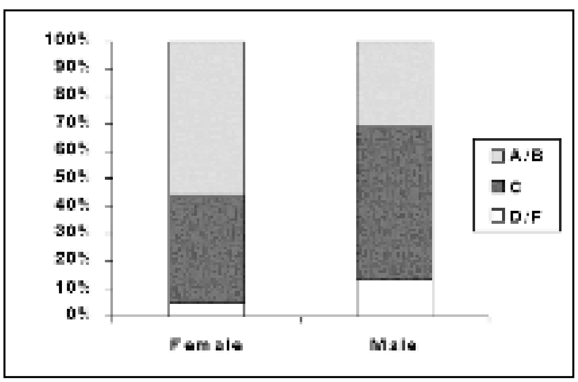

Figure 1. Perceived grades "received."

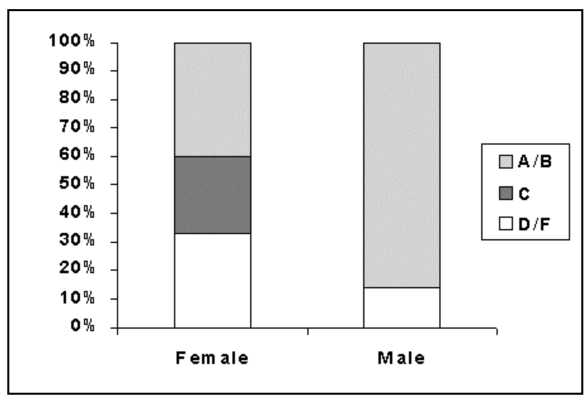

Figure 2. Perceived grades "earned."

Men were more likely to see themselves as having earned high grades and women were more likely to see high grades as received. Of the men reporting grades in "earned" contexts, *none* reported having earned a C. Women who reported grades in "earned" contexts tended to remark most on earning high or low grades. These results are similar to research into gender and mathematics at many levels. Men and boys who do well tend to report that they succeeded because of their own traits (e.g., hard work, intelligence, etc.) and those who fail tend to blame it on external causes (e.g., bad or unfair teacher, unlucky, etc.) In contrast, women and girls tend to ascribe success to external factors and failure to their own internal traits (Gilbert, 1996; Watt & Eccles, 1999).

ABILITY, EFFICACY, AND POTENTIAL IN MATHEMATICS: In essays and interviews students spoke regularly about emotions, including love, hate, fear, frustration, and fury they felt when mathematically engaged. Some students related stories about how their ability self-perceptions influenced career choices.

Maria noted that she had "always loved math and loved teaching it to anyone who would sit still, including my brothers." She recalled a fifth grade teacher who had suggested to her she was gifted: "Mrs. Sitwell said I would be a good 'math mom' and could raise lots of kids to be good in math if I became a teacher." Maria's decision to become a teacher felt like "a calling, you know, like I can do it and can help others to love math too." Meanwhile, a tutoring experience with middle school students during her pre-service teacher program led her to "realize now that sometimes people can do math great, but they don't *want* to do it, they don't enjoy it like I do."

Leo identified a key point in the decline of his perceived mathematical potential in the third grade:

> *I had not labeled Math a problem until it was pointed out, [by] my third grade teacher, that I had a problem with it. Mrs. Roy, my third grade teacher, destroyed my self perception of Math... [she] told me numerous times that I would never do well in Math, and she was right.*

In contrast, Layla wrote in her essay that she always thought of herself as doing well in mathematics but a review of old report cards was a revelation to her:

> *My teacher, Mrs. Wills was one of my favorite teachers. Her comments to my mom and dad were as follows, "Ordinary Numbers, Layla needs to be much more diligent in this area, April 1991." What are "ordinary numbers", and what does that mean? I must not have been a great participator in math in the first grade. That's what I understand from her comments now. Looking back at this report card now I can't believe I was worse than I thought I was, but it was only the beginning.*

Jon, who was efficacious in mathematics as long as fractions were not involved, transferred from college to college as the meeting of a mathematics requirement "reared its ugly head." He reported that his multiple college transfers were based, in part, on the view of his mathematical potential established by him in fourth grade:

> *... the smartest group was called The Dolphins and there were only a handful of kids in this group. Then there were The Sharks, which comprised most of the class.*

Then there were The Whales, the slower kids. Then there was a kid from Arkansas, a kid who liked to start fires, and me. We were Plankton. I'm kidding, we were called The Squids, really, in fact I think I named the group myself.

Jon wrote that his experiences in mathematics "went downhill from there" and his "utter lack of potential in math" led to choosing a degree in theater. He "changed colleges twice and then avoided the required math" until his final semester at university.

DISCUSSION: One valuable result of asking students about their personal mathematical histories is that we can be aware that by the time most students leave high school, their readily recalled memories of mathematical experiences are likely to be evenly split between positive and negative. As one student put it, "every encounter with mathematics carries a risk" – a risk that for many has no more than a 50% likelihood of being associated with a positive outcome.

Students bring to their college mathematics courses the perception that intentional engagement with mathematics is externally driven by factors like grades and the behavioral expectations of instructors. Also, mathematical experiences appear to be related to an intimately felt and emotionally powerful collection self-evaluations about relative ability, efficacy, and potential that are connected to decision-making patterns (including long-term decisions about career).

AFTER COLLEGE, AMONG TEACHERS: Since the report on undergraduate experiences summarized here, I have continued to assign activities like the one in the appendix in undergraduate and graduate courses as well as in professional development with practicing K-12 teachers. For most in-service teachers, writing a mathematical autobiography has turned into writing *two* mathematical autobiographies – one about self as student and one about self as teacher. For a few, all with five or more years of classroom experience, there is a kind of splitting and rejoining of self-as-student and self-as-teacher that occurs in creating the essay. Among these educators, most have made remarks similar to something Pat, a veteran elementary and middle school teacher, wrote,

When I was making lists of math experiences, at first the only way I could organize it was to have my experiences as a learner and then my experiences as a teacher, kind of in a timeline. Then I realized that some of my learning experiences were actually teaching experiences, like when I tutored a classmate in junior high school. And some of my biggest "aha!"s about understanding mathematics came in my first two years in the middle of teaching or planning to teach, or reflecting on something that happened in class.

Similarly, teachers with degrees in mathematics, or many years of advanced mathematical learning, have often echoed Belinda's reflection on her goals for students in her sixth and seventh grade mathematics classes:

> *Though it still sometimes happens, I don't get that math discovery feeling while I'm teaching much anymore. Nowadays, it comes when I take a class or go to a conference presentation. When I put myself in the role of "student" is when it feels most natural to expect to learn some mathematics. At t his point, after 12 years in the classroom, I'm more interested in contributing to students "owning" the math for themselves than I am in creating particularly flashy memories.*

Also, experienced teachers frequently offer synthesizing thoughts at the end of their mathematical autobiographies that are like Antonio's thinking, after six years as a teacher at an alternative high school:

> *I know it may sound cold, but I'm sure I accumulated a whole lot more good mathematical thinking from experiences I don't remember in detail, from the unremarkable yet effective day to day teaching I was lucky enough to experience for most of my schooling. I own my knowledge and I own my ignorance now, which means I am the one who can learn AND teach myself. I think that's the direction I'm going with my math teaching too, helping my students know themselves as learners of mathematics so they can make good decisions about how they learn. So they can be informed consumers about what they take away from what others offer to them in the name of "teaching."*

NOTE ON USING THE MATHEMATICAL AUTOBIOGRAPHY ASSIGNMENT: One major theory of autobiographical memory proposes that the self is actually a collection of "possible selves" including who one has been, is now, and may potentially be in the future (Markus & Nurius, 1986). The results of student interviews and journaling reported by some (Borasi & Rose, 1989; Carlson, 1999; Davis, 1997; Millsaps, 2000; Nimier, 1993; Yow, 2012) may be more representative of current-self view than past- self re-view. Extended reflective or expressive writing is likely to yield distinctions between current-self and recollective reporting (e.g., Brandau, 1988). The mathematical autobiography assignment used for this report included prompts for reflective memory searching and was part of the course grade.

Does the use of the mathematical autobiography as a curricular extension activate useful self-reflection? The answer seems to be: "Yes, for some students." The mathematical autobiography is an expressive writing exercise and appears to foster a useful sense of perspective. Half of those interviewed remark on feeling more self-control around mathematics by allowing themselves to write about and feel emotions without getting tangled up in them.

Instead of seeking to rid students of their reactions to mathematics, a mathematical autobiography allows students to acknowledge their responses and build reflective awareness of them. Self-regulation may come in many forms: from response to the anticipation of consequences of behavior choices (Boekaerts, 2000), to self-aware constructing of goals (Locke & Latham, 1990), to beneficial inner-speech.

However, it must be noted that such self-regulation requires an appropriately supportive classroom. Without in-class prompts and opportunities that encourage personally owned, self-reliant accomplishment in mathematics, the most significantly impacted students may continue to attribute their successes to external rather than personal, internal, sources (Bandura, 1997; Borasi & Rose, 1989; Brandau, 1988; Pajares & Schunk, 2002).

Developmentally, the kind of self-reflection and recollective thought called for in the assignment is only deeply possible among late adolescents (Rubin, 1996). While younger students can benefit from such an assignment, it may be most productive for students in middle school and higher. At all age levels, the teacher can learn plenty from reading what students write! Can the assignment harm rather than help? Research on expressive writing indicates that such an assignment is unlikely to be a hindrance to learning (Hirsch & King, 1983). Among the 300+ students who wrote essays during this study, in no case did the assignment ever appear to have a detrimental effect (though a few did complain about "having to write in a math class!?!").

What people offer in their mathematical autobiographies are real memories; whether or not they are precise and fully accurate memories of real events may be debatable (Weingardt, Loftus, & Lindsay, 1995). However, it may not matter which they are. Memories of both types shape the way a person perceives experience, conceives of the world, regulates thoughts and emotions, and interacts with others.

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APPENDIX

THE ASSIGNMENT WEB PAGE

MATHEMATICAL AUTOBIOGRAPHY PROJECT
PLEASE READ THIS ENTIRE PAGE!

Preliminary step: Make a list of TWENTY mathematical experiences. For example, what can you recall of learning to count?... of learning to tell time...? of learning what fractions mean?...of learning how to use money? Each person should reach as far back into her/his personal history as possible. Review old report cards; talk to friends, parents, siblings, caretakers, etc. to collect information, anecdotes and experiences. Does your recollection of grades in your mathematics courses match the actual grades on your old report cards?
[You might be surprised.]

Draft step: Write a rough draft of at least 850 words (type it, double-spaced) using at least five of the experiences from the list you generated. It is probably best to write it on a computer (and save it to a disk) so that you can edit later and so that you can use the word-count utility most word-processing programs have!

The assignment: Referring to your rough draft and the list generated in the first step, write an essay of 1100 to 3000 words which relates some of the 20 experiences (at least five) in detail. Discuss how those experiences have influenced current attitudes, feelings, thoughts about mathematics and life goals. Include names, locations. For example: “When I was in the ninth grade at Norco High School (that's in Riverside County in Southern California) I had an Algebra teacher named Miss Trimble who sometimes had us do math outside. One incident I recall vividly was the warm, sunny day the whole class went to the football field and we..."

The essay will be graded as follows:
10 points for length: if the paper is less than 1100 words then the length score will be reduced; the scores for grammar and content will be proportionally reduced as well 15 points for spelling and grammar. 75 points for content: as long as the paper is coherent, is about the student's personal math history and is at least 1100 words long, all content points will be earned. The instructor is happy to proofread drafts of the paper during office hours.

DIGITAL ART AND ANIMATION MOTIVATES SCHOOL STUDENTS TO PERSIST IN SOLVNG STEM PROBLEMS[1]

SANDI LAVITO, STUDENT, CALIFORNIA STATE UNIVERSITY STANISLAUS

INTRODUCTION: For over fifteen years, the High School Mathematics Access Program (HiMAP) has enhanced learning opportunities for 6th-12th grade students in the California State University, Stanislaus service area. HiMAP is an academic assistance and enrichment program that offers young students tutoring and coaching in mathematics, physics, chemistry, biology, general science and technology. This program, launched in 1995, has been a consistent fixture in the academic landscape of the university and is constantly being improved and innovated.

During the 2011- 2012 academic year HiMAP included yet another innovation to the curriculum—a digital art and animation module. This was designed as a way for students to apply art and animation techniques to better understand solve mathematical and scientific concepts.

Additionally, the animation program was used as an incentive to motivate students to work harder in the core areas of mathematics and science in the HiMAP program. Students who finished the assigned homework and answered questions generated in the classroom correctly and in a timely manner were rewarded with the opportunity to learn how to create animations—a skill few young students can claim. Although participation in the art and animation module was largely treated as a reward, it emerged as a way to bolster confidence in students whose interest in STEM curriculum was not high.

IMPLEMENTATION OF THE PROGRAM: The animation program emphasized both traditional and digital key artistic elements dealing with proportions; they are not allowed to use preset brushes. These elements were addressed across the curriculum through the following seven lessons:

1. *Learning Basic Terminology*
2. *Basic Character Design*
3. *The Turn Around*
4. *Character Expressions*
5. *Movement Cycles*
6. *The Digital Toolbox*
7. *The Final Animation*

[1] This paper was written under the tutelage of Viji Sundar, Professor of Mathematics, CSU Stanislaus.

Here is a brief description of each lesson:

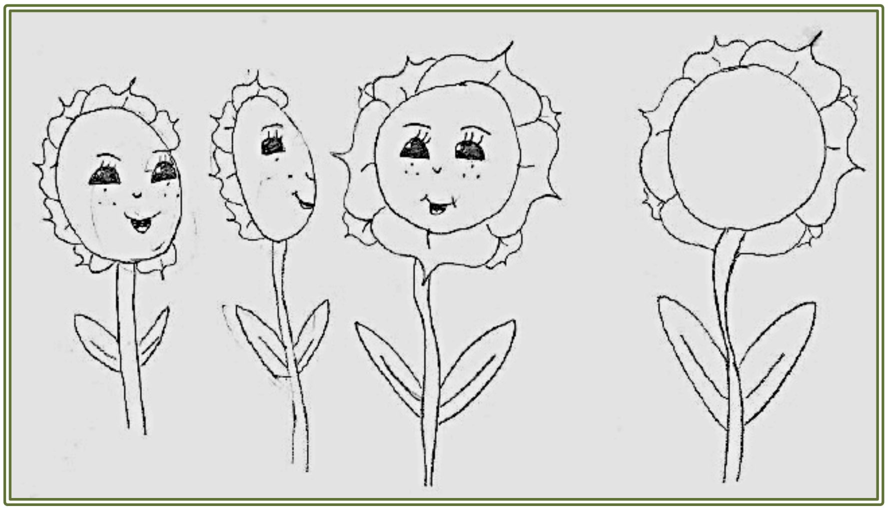
1. Learning Basic Terminology: In the first of these lessons, the general process of animation was explained to students. Unfamiliar terms such as "silhouette" and "frame rate" were explained in depth. We observed that students who picked up quickly on the overall process of animation were also in a better position to understand how the art and animation lessons could be used in mathematics and science.

2. Basic Character Design: The second activity was more hands on: students were instructed to design a basic "character." Characters ranged from inanimate objects to humanoid characters. Students were taught that simple character designs are easier to animate and simplicity would be an advantage in later activities.

3. The Turn Around: We noted that some students still overestimated how much time and care and patience is required in animation; some created overly complicated characters despite all advice to the contrary. But, rather than curbing creativity, students were allowed to learn the cost of their decision in the "Turn Around" activity. Once a character was created, students would then create a "Turn Around" for that character. A "Turn Around" is a reference sheet used by animators for a reference sheet where a single character is represented in multiple views lined up side by side to maintain proportions. For the sake of this lesson the views were limited to front, back, and three-quarter turn. The students used this reference sheet as a way to exercise spatial awareness. In order to draw characters in believable ways, they had to ask themselves many questions. For example, a student drawing an anthropomorphic2 lollipop would have to ask: What does a cylinder look like when it is bent? What does that bent cylinder look like from a lateral view? From the front? The goal of this exercise was to encourage students to actively observe phenomena they experience each day and to deconstruct these movements logically—an approach similar to that of analyzing a STEM problem in order to visualize these various physical properties so that a "Turn Around" could be created, various blocks, spheres and other three-dimensional objects were made available to the students in the form of paper, plastic, and wooden blocks. This allowed students to physically touch and manipulate shapes they might not yet be able to spatially visualize. Figure 1 is a Turn Around created by one of the students who used shapes to understand how three-dimensional shapes behaved. The image shows how the student was able to keep the character in proportion across multiple images and to maintain realistic movements.

[2] Ascribing human forms and attributes to the characters on paper.

FIGURE 1: ONE STUDENT'S SAMPLE WORK DEMONSTRATING THE TURN AROUND TECHNIQUE OF A SUNFLOWER CHARACTER'S POSITIONS

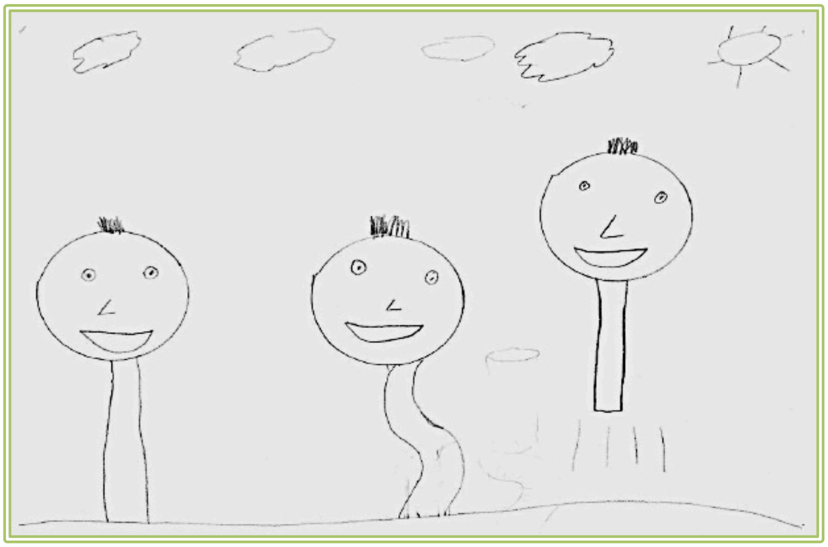


4. CHARACTER EXPRESSIONS: Following the Turn Around lesson, further activities were assigned to help students learn to deconstruct large, complex problem systems in order to understand the overall structure of the project. The complex problem set for the students were sets of questions related to just how the character should look, should act, how movements actually work, and, in general, how to make the image created exhibit sympathetic human qualities.

The fourth task called for students to create reference shots of their character expressing various emotions. As predicted, simple emotions such as happiness or sadness were easy for students to represent, but others such as fear or exasperation were more difficult. For those more difficult emotions, students were paired up and were instructed to observe the facial movements of their emoting partners. The exercise enabled the students to articulate what differentiated a genuine expression from a spurious one. This added another layer of sophistication to their problem solving skills in that in encouraged an attention to minute details.

5. MOVEMENT CYCLES: The ability to deconstruct actions we conjecture is in many ways similar to deconstructing a mathematics or physics problem. This highlighted in the fifth activity: students were to create the key frames (or main images) of various cycles. In animation, "a cycle" is a set of unique actions that, that when repeated, creates the illusion of motion. In this lesson, the basic cycles the students were taught to draw were walking, running, and jumping. These cycles represented a range from simple (jumping, three frames) to increasingly complex (running, 15 frames). Figure 2 illustrates the artwork of one student showing a jump cycle featuring a lollipop character of his own design.

FIGURE 2: JUMP CYCLE FEATURING A LOLLIPOP CHARACTER



Initially, students did not appreciate just how complicated such movements truly are, simply because they can perform these actions without consciously moving each individual muscles and have been doing so almost all of their lives. To help them deconstruct the movements, one partner was again asked to 'act out' a particular action in slow motion while the other partner took notes. Even though the students' drawing abilities saw only slight improvement, they learned to not take the simplicity of something for granted. Many expressed astonishment when they realized how much thought and scientific understanding supported even the most simplistic of animations. It was a problem solving lesson well learned!

6. THE DIGITAL TOOLBOX: The final lesson of the art and animation program was to have students work with a computer in a capacity outside of the standard "word processor" they may or may not have learned in school. Many of the students did not have experience using "a set of tools" or a "toolbox approach" that is common in this type of computer software and allows for ready-made or already-mastered tools. A toolbox approach enables students to learn about the available tools without being trapped into repeating the same elementary steps. This process is similar to using a formula in solving a mathematics problem; e.g. using the quadratic formula to find the roots of a second degree polynomial.

7. THE FINAL ANIMATION: For the final component of the module, the students were asked to create small animations using these tools in any manner they saw fit. Many of these animated Graphics Interchange Format (GIF) files could be formatted so that the animation shuffles through layers of figures giving the illusion of movement; these movements were less than ten frames long and lasted between one-to-five seconds. This allowed students to use a mouse or familiarize themselves with the concept of keyboard short cuts to create their own animations.

CONCLUSION: The most important goal of the HiMAP Art and Animation component was to increase the confidence in students who participated in this program. The students were all capable of solving problems but many lacked some confidence in their own abilities, compounded by shyness. However, they were able to figure out how inanimate shapes can be manipulated in two-dimensions to create live animations using the appropriate computer software. We conclude from this experience that given the right tools and environment, students will learn from their mistakes and will persist in completing the task thus gaining more confidence in themselves and in problem solving. The animation enrichment program nurtured these students' budding self-confidence.

Additional Resources:

Cartoonster. http://www.kidzdom.com/tutorials/.
FluxTime Studios. http://www.fluxtime.com/.
Teaching Ideas. "Animation Ideas".
http://www.teachingideas.co.uk/ict/animationideas.htm.

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California State University, Stanislaus

One University Circle

Turlock, CA 95382

California Mathematics Project